Efficient inverse radiation analysis in a cylindrical geometry using a combined method of hybrid genetic algorithm and finite-difference Newton method

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Received 6 October 2006; received in revised form 22 April 2007; accepted 3 May 2007

Abstract

An inverse radiation problem was considered to estimate boundary conditions such as temperature distribution and emissivity in axisymmetric absorbing, emitting, and scattering medium, given the measured incident radiative heat fluxes. The finite-volume method was employed to solve a direct radiative transfer equation for a two-dimensional axisymmetric geometry. Various parameter estimators, such as conjugate-gradient method, hybrid genetic algorithm, and finite-difference Newton method, were employed to solve the inverse problems, while discussing their performances in terms of estimation accuracy and computational efficiency. Based on this, we proposed, as a best inverse analysis tool, a new combined method that adopted the hybrid genetic algorithm as an initial value selector and used the finite-difference Newton method as a parameter estimator.

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Keywords: Hybrid genetic algorithm; Conjugate-gradient method; Finite-difference Newton method; Inverse radiation boundary problem; Parameter estimation

1. Introduction

Inverse radiation analysis has been concerned with the estimation of radiative properties from measured radiation quantities [1,2]. The determination of medium properties such as extinction coefficient, absorption coefficient, single scattering albedo, phase function, optical depth, and gas temperature as well as surface properties such as emissivity and boundary temperature has been achieved by inverse radiation analysis from measured intensities or temperatures [3–7]. A review of the references for inverse radiation analysis in 1986–1991 is available in the literature [8]. Recently, the application of inverse radiation problems has been extended to multi-dimensional cases [4–6].

Especially, inverse radiation analysis in an axisymmetric medium has been conducted to estimate the source term distribution or radiative properties inside a medium in the parameter estimation approach. Li [5]...
## Nomenclature

- **d**: a vector of the direction of descent
- **D**: 
  - direction weights
- **\( \vec{\varepsilon}_i \)**: unit vector normal to direction \( i \)
- **I**: radiation intensity, \( \text{W}/(\text{m}^2 \text{sr}) \)
- **\( I_b \)**: blackbody radiation intensity, \( \text{W}/(\text{m}^2 \text{sr}) \)
- **L**: cylinder height, m
- **\( N_{\text{gen}} \)**: generation number
- **\( N_{\text{gen, max}} \)**: maximum generation number
- **nm**: number of measurement points
- **np**: number of unknown parameters
- **P**: a vector of unknown parameter
- **q**: incident radiative heat flux, \( \text{W}/\text{m}^2 \)
- **\( \vec{r} \)**: position vector of intensity
- **\( r_c \)**: cylinder radius, m
- **\( r_{ij} \)**: correlation coefficient
- **S**: radiative source term
- **\( s \)**: direction vector of intensity
- **\( T_g \)**: temperature of participating medium, K
- **\( T_i \)**: boundary temperature at surface \( i \)
- **X**: sensitivity matrix
- **\( X_s \)**: sensitivity coefficient

### Greeks symbols

- **\( \beta \)**: search step size
- **\( \beta_o \)**: extinction coefficient, \( = \kappa_a + \sigma_s \text{ m}^{-1} \)
- **\( \gamma \)**: conjugation coefficient
- **\( \Delta A_p \)**: \( \Delta V \) with surface area and volume of the control volume
- **\( \Delta \Omega \)**: control angle
- **\( \varepsilon_i \)**: boundary emissivity at surface \( i \)
- **\( \theta \)**: polar angle measured from the \( z \) direction, rad
- **\( \kappa_a \)**: absorption coefficient, \( \text{m}^{-1} \)
- **\( \sigma_s \)**: scattering coefficient, \( \text{m}^{-1} \)
- **\( \sigma_{st} \)**: standard deviation of measurement data
- **\( \Phi \)**: scattering phase function
- **\( \varphi_o \)**: space variable in the azimuthal direction measured from the \( x \)-axis, rad, Figs. 1 and 2(b)
- **\( \varphi_{\Omega} \)**: angular variable in the azimuthal direction measured from the \( x \)-axis, rad, Figs. 1 and 2(b)
- **\( \xi \)**: uniform random variable
- **\( \zeta \)**: standard normal distribution random variable

### Subscripts

- **E,W,N,S,T,B**: index of neighboring nodal points
- **e,w,n,s,t,b**: index of control volume surfaces
- **e**: estimated value
- **m**: measured value
- **P**: calculating nodal point
- **w**: wall
estimated the source term, i.e. temperature distribution inside a medium, while Ou and Wu [4] simultaneously
determined various radiative properties. For both cases, the boundary wall was assumed to be transparent.

To implement an inverse analysis, first of all, a stable and robust numerical inverse method is needed. To
achieve it, many optimization techniques have been adopted, as an iterative regularization method, to cope
with the ill-posed characteristic of the inverse problem. The conjugate-gradient descent method (CGM) or
steepest descending method, which is one of the gradient-based optimization methods, is usually adopted to
minimize the objective function, which is expressed by the sum of square errors between estimated and
measured data [5,6]. Recently, a search-based optimization method such as the genetic algorithm (GA) has
received much attention for its outstanding characteristics, for instance, less dependence on initial value and
no need of gradient information, especially in nonlinear or multi-parameter problems [3,7,9]. Additionally,
Yang [10] adopted the quasi-Newton method, which is originally a solver for a set of nonlinear equations, for
the inverse conduction problem. In this method, the errors between estimated and measured data are
expressed by a set of equations as many as the number of measurement points.

Comparison of various inverse methods has been carried out by many researchers [9,11]. Among others,
Orain et al. [9] has compared the gradient-based method with the search-based method, especially in a highly
correlated case between parameters. The GA as well as the Gauss method was adopted as one of the search-
based methods together with the gradient-based method. The GA yielded a better solution when the
parameters were highly correlated.

In this study, we consider a highly correlated parameter estimation for the five test cases. The gray, diffusely
emitting, and reflecting wall is assumed for the estimation of boundary conditions such as non-uniform
temperatures with constant emissivity. The results are compared and discussed to determine which method
among CGM, finite-difference Newton method (FNM), and hybrid genetic algorithm (HGA) is preferred for
a highly correlated case. Finally, a combined method of HGA and FNM is suggested for better results.

2. Analysis
2.1. Model description

Fig. 1 shows an axisymmetric enclosure representing a simplified combustor and its coordinate system
which is filled with an absorbing, emitting, and scattering gray medium, with $\kappa_a = 0.5 \text{ m}^{-1}$, $\sigma_a = 0.5 \text{ m}^{-1}$, and
$T_g = 1000 \text{ K}$. The length and radius of the cylinder are 4 m and 1 m, respectively. The walls are diffusely
emitting and reflecting gray walls, while surfaces 1 and 3 have an emissivity of 0.8 and temperatures of
600 and 800 K, respectively. Boundary conditions such as temperature and emissivity of surface 2 are
unknown, and will be estimated simultaneously when incident radiative heat flux is available in this analysis.
Conventionally, the boundary temperature can be measured using thermocouples, and naturally some errors
are contained in the measured data. Also, it is more difficult to measure emissivity experimentally. For this
reason, we try to simultaneously estimate boundary temperature distribution and uniform emissivity based on
measured incident radiative heat fluxes using inverse analysis. The spatial and angular domains are discretized
into $20 \times 50$ control volumes and $8 \times 14$ control angles to get an accurate radiative intensity distribution at
surface 2.

2.2. Radiative transfer equation (RTE)

The RTE governing radiation intensity for a gray medium at any position $\vec{r}$ along a path $\vec{s}$ through an
absorbing, emitting, and scattering medium is given by
\[ \frac{dI(\vec{r}, \vec{s})}{ds} + \beta_o I(\vec{r}, \vec{s}) = S(\vec{r}, \vec{s}), \]

where \( \beta_o = \kappa_a + \sigma_s \) and

\[ S(\vec{r}, \vec{s}) = \kappa_a I_b(\vec{r}) + \frac{\sigma_s}{4\pi} \int_{4\pi} \Phi(\vec{s}', \vec{s}) I(\vec{r}, \vec{s}') d\Omega'. \]

Here, \( \Phi(\vec{s}', \vec{s}) \) is the scattering phase function for radiation from the incoming direction \( \vec{s}' \) to the scattered direction \( \vec{s} \) and it is approximated by a finite series of Legendre polynomials as

\[ \Phi(\vec{s}', \vec{s}) = \Phi(\cos \Psi) = \sum_{j=0}^{J} C_j P_j(\cos \Psi), \]

where \( C_j \)'s are expansion coefficients and \( J \) is the order of the phase function. We assumed isotropic scattering in this study.

The boundary condition for a diffusely emitting and reflecting wall can be written as follows:

\[ I(\vec{r}_w, \vec{s}) = \varepsilon_w(\vec{r}_w) I_b(\vec{r}_w) + \frac{1 - \varepsilon_w(\vec{r}_w)}{\pi} \int_{\vec{s}' \cdot \vec{n}_w < 0} I(\vec{r}_w, \vec{s}') |\vec{s}' \cdot \vec{n}_w| d\Omega', \]

where \( \varepsilon_w \) is wall emissivity and \( \vec{n}_w \), which has a positive value when the ray travels from the wall to the medium, is the unit normal vector to the wall.
The incident radiative heat flux is calculated by integrating the intensities over all outgoing solid angles as follows:

\[ q_e(\vec{r}_w) = \int_{\hat{s} \cdot \hat{n}_w < 0} I(\vec{r}_w, \vec{s}) |\vec{n}_w \cdot \vec{s}| \, d\Omega. \]  

(4)

2.3. Finite-volume method (FVM) for radiation

In solving direct problems, the FVM is adopted for its convenience in selecting the solid angle while guaranteeing an exact global conservation of radiative energy. Chui’s way to treat the axisymmetric geometry is employed [12,13]. To obtain the discretization equation, Eq. (1) is integrated over a control volume, \( \Delta V \), and a control angle, \( \Delta \Omega^m \), in the axisymmetric orthogonal grid as shown in Fig. 2. By assuming that the magnitude of the intensity in a control volume and control angle is constant, the following finite-volume formulation can be obtained:

\[ \sum_{i=e,w,n,s,t} I^m_i \Delta A_i \Delta \Omega^m_i = (-\beta_{o,p} I^m_{ip} + S^m_{e,p}) \Delta V \Delta \Omega^m, \]  

(5a)

where

\[ D^m_{ci} = \int_{\phi^m_i}^{\phi^m_{i+1}} \int_{\theta^m_i}^{\theta^m_{i+1}} (\vec{s} \cdot \vec{n}_i) \sin \theta \, d\theta \, d\phi, \]  

(5b)

\[ \vec{s} = \sin \theta \cos \phi \vec{e}_x + \sin \theta \sin \phi \vec{e}_y + \cos \theta \vec{e}_z, \]  

(5c)

\[ \vec{n}_e = \sin \phi \vec{e}_x - \cos \phi \vec{e}_y, \]  

(5d)

\[ \vec{n}_w = -\sin \phi \vec{e}_x + \cos \phi \vec{e}_y, \]  

(5e)

Fig. 2. (a) Angular control angle and (b) spatial control volume.
\( \vec{n}_n = -\vec{n}_s = \cos \phi_{s,n} \vec{e}_x + \sin \phi_{s,n} \vec{e}_y, \) 
\( (5f) \)
\( \vec{n}_t = -\vec{n}_b = \vec{e}_z, \) 
\( (5g) \)
\[ S'^m_r = \kappa I_b + \frac{\sigma_b}{4\pi} \int_{\Omega'=4\pi} I'' m \Phi_{m' \rightarrow m} \, d\Omega', \] 
\( (5h) \)
\[ \Delta \Omega'^m = \int_{\phi'^m}^{\phi'^{m+}} \int_{\theta'^m}^{\theta'^{m+}} \sin \theta \, d\theta \, d\phi. \] 
\( (5i) \)

To relate the intensities on control-volume surfaces to a nodal one, the step scheme, which is not only simple and convenient, but also ensures positive intensity, is adopted. Then, the final discretized equation for FVM is obtained by
\[ a_m I^m = \sum_{I=E,W,S,N,T,B} a_I^{m} I'^m_{I} + b_p^m, \] 
\( (6a) \)
\[ a_I^{m} = -\Delta A_i D_{ci, in}^{m}, \] 
\( (6b) \)
\[ a_p^m = \sum_{I=E,W,S,N,T,B} \Delta A_i D_{ci, out}^{m} + \beta_n \Delta V \Delta \Omega'^m, \] 
\( (6c) \)
\[ b_p^m = (S'_{r,p}) \Delta V \Delta \Omega'^m. \] 
\( (6d) \)

where
\[ D_{ci, out}^{m} = \int_{\Delta \Omega^m} (\vec{n}_i \cdot \vec{s}) \, d\Omega, \quad \vec{n}_i \cdot \vec{s} > 0, \] 
\( (6e) \)
\[ D_{ci, in}^{m} = \int_{\Delta \Omega^m} (\vec{n}_i \cdot \vec{s}) \, d\Omega, \quad \vec{n}_i \cdot \vec{s} < 0. \] 
\( (6f) \)

2.4. Conjugate-gradient descent method (CGM)

The CGM is a straightforward and powerful iterative technique for solving nonlinear as well as linear inverse problems of parameter estimation. This method has been adopted by many researchers in dealing with inverse radiation [5,6].

Once this method sets one candidate solution, it searches the local optimum point along the descent direction by minimizing the objective function. Whereas the steepest descending method, one of the gradient-based methods, determines the descending direction only with the gradient information of the current iteration so that sometimes it makes convergence slower, the CGM makes use of the present as well as previous descending history so that convergence becomes faster and more stable.

The iterative procedure for parameter estimation is as follows:
\[ \mathbf{P}^{k+1} = \mathbf{P}^k - \beta^k \mathbf{d}^k, \] 
\( (7) \)
where \( \mathbf{P} \) is a vector of unknown parameters, \( \beta \) is search step size, \( \mathbf{d} \) is direction of descent, and superscript \( k \) is iteration number. The direction of descent is determined by the combination of gradient direction and direction of descent in the previous iteration.
\[ \mathbf{d}^k = \nabla S(\mathbf{P}^k) + \gamma^k \mathbf{d}^{k-1}. \] 
\( (8) \)
Here, the expression of conjugation coefficient is given by

$$g_k = \frac{\sum_{j=1}^{np} \left[\nabla S(P^k)\right]_j^2}{\sum_{j=1}^{np} \left[\nabla S(P^{k-1})\right]_j^2} \quad \text{with} \quad g^0 = 0,$$

(9)

where $np$ is the number of unknown parameters.

The objective function is denoted by

$$S(P^k) = \sum_{i=1}^{nm} (q_{m,i} - q_{e,i}(P^k))^2.$$  

(10)

The $j$ component of the gradient of objective function is given by

$$[\nabla S(P^k)]_j = -2 \sum_{i=1}^{nm} X_{j,i} [q_{m,i} - q_{e,i}(P^k)],$$

(11)

where $nm$ is the number of measurement points, $X_{j,i}$ is the sensitivity coefficient defined as the first derivative of the estimated heat flux at $i$ measurement position, $q_{e,i}$ with respect to the unknown parameter, $P_j$, that is,

$$X = X_{j,i} = \frac{\partial q_{e,i}(P^k)}{\partial P_j},$$

(12)

and $q_{m,i}$ is the measured heat flux at $i$ measurement position.

Finally, the search step size is calculated by minimizing the function $\nabla S(P^{k+1})$ with respect to $b_k$, which becomes after some manipulation

$$b^k = \frac{\sum_{i=1}^{nm} \left( \sum_{j=1}^{np} X_{j,i} d^k_j \right) [q_{e,i}(P^k) - q_{m,i}]}{\sum_{i=1}^{nm} \left( \sum_{j=1}^{np} X_{j,i} d^k_j \right)^2}.$$  

(13)

A detailed description on how to apply CGM to inverse heat transfer problems can be found in the literature [2].

### 2.5. Finite-difference Newton method (FNM)

Many methods exist for solving a set of equations when the number of equations is the same as that of unknown variables. Among others, open methods, such as Newton’s method and the secant method, are well known for quicker convergence than bracketing methods, such as the bisection method [14].

In applying Newton’s method to inverse analysis, instead of the least square form, a new set of equations has to be formulated using the difference between the estimated and measured data as follows:

$$S_i(P) = q_{e,i}(P) - q_{m,i} \quad \text{for} \quad i = 1, \ldots, nm.$$  

(14)

In the above equation, $S$ can be expressed in Taylor series, and should be zero to find appropriate parameters such that

$$S(P) \approx S_0(P^0) + \nabla S(P)(P - P^0) = 0,$$

(15)

where $\nabla S(P) = \partial S(P)/\partial P = \partial q_e(P)/\partial P$, which is eventually the matrix of sensitivity coefficients, $X$, as defined in Eq. (12). Because a finite-difference approximation is used to calculate the sensitivity coefficient, this method is called the FNM [15].

After some manipulation of Eq. (15), we can derive the following relation:

$$P^k = P^{k-1} - X^{-1} S.$$  

(16)

Here, it must be noted that the above relation may be applied only for the case where the number of equations and unknown variables is the same. However, based on the fact that the number of equations can be greater than that of unknown variables in the inverse analysis, Eq. (16) is modified
as follows [15]:
\[ P^k = P^{k-1} - \left[ X^T X \right]^{-1} X^T S. \] (17)

2.6. Hybrid genetic algorithm (HGA)

The GA, based on the concept of natural selection, is a robust global parameter estimator [16,17]. It represents and manipulates candidate solutions in the genotype, and each candidate solution group, which corresponds to the vector of unknown parameter, \( P \), in Eqs. (7) and (17), is called an individual. As generation number increases, individuals randomly generated at the first stage approach the global optimum through its main operators such as selection, crossover, and mutation so that there is no influence of initial values.

Fig. 3 shows the flowchart of an HGA used in this study. At the first initialization stage, an initial population, which is a set of individuals, is randomly generated in design space as shown in Fig. 4, and each individual forms an array of candidate solutions that are represented in binary or float-point representation as shown in Fig. 5. In this study, the population size is fixed to 10 to reduce the computational time, which is the main drawback of the GA. Furthermore, the float-point representation is used to reduce the length of the chromosome to the number of design variables.

The fitness of each individual must be evaluated to determine which individual survives for the next generation and which individual ends its life in this generation. Evaluation of the fitness is done by calculating the objective function, Eq. (10), with each individual’s candidate solutions. Since the evaluation is carried out by fitness value, gradient information is not needed for optimization in GA.

Through the selection operation, a fitter individual among individuals of the present generation is selected to reproduce offspring for the next generation. There are various selection schemes such as fitness-proportional selection, ranking selection, and tournament selection. In this study, fitness-proportional selection is adopted so that a fitter individual with higher probability could be selected. However, when the population size is small, individuals of the next generation are likely to be filled with super-individuals that are superior to the other individuals so that sometimes the individuals fall into the local optimum in the end. Not

Fig. 3. Flowchart of hybrid genetic algorithm.
only to prevent it, but also to keep the diversity of individuals, stochastic universal sampling, shown in Fig. 6, is used for selection operator.

Individuals chosen by selection operation undergo the operation of crossover and mutation for the reproduction of fitter individuals. In crossover operation, individuals, which are as many as the number corresponding to the probability of crossover, meet their mates and exchange their genes. In this study, the one-point crossover, shown in Fig. 7, is used, and thus two individuals swap their genes at one crossing point. The mutation operator allows some genes to change their values within the design space to find fitter individuals. The probability of mutation controls the number of genes undergoing the mutation operation. The non-uniform mutation is adopted here to improve the fine local tuning ability as the generation number increases [16].

If 
\[ s = \langle P_1, P_2, \ldots, P_k, \ldots, P_{np} \rangle \]

is the selected chromosome at a generation number and gene \( P_k \) is chosen for mutation operation, after mutation, the resulting chromosome becomes 
\[ s = \langle P_1, P_2, \ldots, P'_k, \ldots, P_{np} \rangle, \]

where
\[
P'_k = \begin{cases} 
P_k + \Delta(t, UB - P_k) & \text{if } \xi \geq 0.5, \\
P_k - \Delta(t, P_k - LB) & \text{if } \xi < 0.5, 
\end{cases}
\]

and LB and UB are lower and upper bounds for the gene \( P_k \). The function \( \Delta(t,y) \) returns a value in the range of \([0,y]\) such that the probability of \( \Delta(t,y) \) being close to 0 increases as \( t \) increases, and the following function is
where $b$ is a system parameter for determining the degree of dependency on generation number, $t$ ($b = 1$ here).

After mutation, the elite strategy is used to ensure a monotonic improvement by copying the best individual of the present generation to the next generation.

A local optimization algorithm (LOA) is often included in the GA to overcome disadvantages such as the inability of the fine local tuning; hence this type of GA is called the HGA. LOA makes individuals move from near optimal range to optimal point more quickly. In this study, an operator used for non-uniform mutation is adopted for LOA. Usually, the gradient-based optimization technique is used for LOA. However, to maintain the stochastic feature of the GA, a search-based technique is adopted. After determining the elite individual in the elite strategy, LOA is applied only to the elite individual to reduce computational time as well as to maintain the diversity of individuals. A total generation number of 100 is used, and detailed behavior of HGA used in this study is found in the literature [7].

2.7. Sensitivity and correlation analysis

If we know the possibility of simultaneous estimation of parameters before the execution of inverse analysis, it would be time saving. In that respect, the sensitivity and correlation analysis provides useful information. Sensitivity coefficients denote a measure of the effects of changes in unknown parameters, $P_i$, on estimated radiative heat flux value, $q_e$ [2]. A small magnitude of sensitivity coefficients indicates that estimation of that parameter would be very difficult, because the same radiative heat flux value would be obtained in a wide range of values of $P_i$. Also, when the sensitivity coefficients of each parameter are linearly dependent, simultaneous estimation of these parameters would also be very difficult [18]. In this study, for sensitivity analysis, the following relative sensitivity coefficients are used, because the order of magnitude between parameters is different:

$$X^*_{j,i} = P_j \frac{\hat{q}_{e,i}(P^k)}{\hat{P}_j}.$$  \hspace{1cm} (20)

In order to calculate the sensitivity coefficient, finite-difference approximation is used:

$$\frac{\hat{q}_{e,i}}{\hat{P}_j} \approx \frac{q_{e,i}(P_1, \ldots, P_i + \epsilon P_i, \ldots, P_{np}) - q_{e,i}(P_1, \ldots, P, \ldots, P_{np})}{\epsilon P_j}$$

for $i = 1, \ldots, nm$ and $j = 1, \ldots, np$, \hspace{1cm} (21)

where $np$ is the number of parameters to be estimated, $nm$ is the number of measurement points, and $\epsilon$ is a very small number.

When the number of parameters to be estimated is more than 3, a decision of linear dependency of sensitivity coefficients is not clear, and also its magnitude of linear dependency is more difficult to determine.
In this case, the correlation analysis can determine the feasibility of simultaneous estimation. The correlation coefficient between two parameters \(i\) and \(j\) is expressed with the sensitivity coefficient as follows:

\[
 r_{ij} = \frac{\Phi_{ij}}{\sqrt{\Phi_{ii} \Phi_{jj}}} \text{ for } i, j = 1, \ldots, np, \tag{22}
\]

where \(\Phi = [X^T X]^{-1}\) and \(X\) is the matrix of sensitivity coefficients.

If the value of correlation coefficient for a pair of parameters is greater than 0.9, the parameters are considered to be highly correlated so that a simultaneous estimation becomes very difficult when the gradient-based method is adopted for an estimator [9].

2.8. Inverse analysis procedure

In this study, boundary conditions such as temperature distribution and emissivity of the side wall, namely, \(T_2\) and \(\varepsilon_2\), are unknown, while the other quantities are all known. The approach for their estimation becomes different, depending on the method adopted.

For the CGM and HGA, an estimation of unknown boundary conditions is achieved by a minimization of the objective function defined in Eq. (10), which is expressed by the square sum of errors between estimated and measured data. In the no measurement error case, when the value of objective function reaches a very small number, the computational procedure is terminated, while the discrepancy principle is adopted as the stopping criterion in the measurement error case; thus the value of the objective function should satisfy the condition \(S(P^k) < nm \times \sigma^2\).

For the FNM, the termination of procedure is exercised according to values of a set of equations, Eq. (14). As the difference in the values of equations between the present and the previous iteration becomes small, the estimation of result is considered to converge.

3. Results and discussion

3.1. Sensitivity and correlation coefficient analysis

Five test cases have been considered for the comparison of various inverse methods. Table 1 lists the test functions and parameters to be estimated for each case. Cases 1–3 are related to the parameter estimation of constant values, while cases 4 and 5 are related to determine the coefficients of polynomial function. Especially, case 5 belongs to the simultaneous estimation of temperature distribution and emissivity. The main objective of this study is to estimate the boundary condition of surface 2 only. However, cases 1 and 2 are included to investigate the influence of correlation coefficient on estimation accuracy.

The exact values of temperature and coefficients of polynomial function listed in Table 1 are ones divided by 1000 to match with emissivity value. Fig. 8 illustrates a variation of the sensitivity coefficients of cases 1 and 5 along the axial length of the cylinder. Usually, the higher the sensitivity coefficient, the easier the determination of the corresponding parameter. Just based on the figure, it is hard to figure out how much some parameters are linearly dependent on each other so that the correlation analysis is essential. Table 2 shows the correlation coefficients of all the pairs of each parameter. For cases 3–5, some correlation
Fig. 8. Relative sensitivity coefficient for cases 1 and 5.

Table 2
Correlation coefficients of all the pairs of parameters for each case

<table>
<thead>
<tr>
<th>Case</th>
<th>Pairs of parameters</th>
<th>Correlation coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\varepsilon_1-\varepsilon_2, \varepsilon_1-\varepsilon_3, \varepsilon_2-\varepsilon_3$</td>
<td>$-0.905, 0.799, 0.824$</td>
</tr>
<tr>
<td>2</td>
<td>$T_1-T_2, T_1-T_3, T_2-T_3$</td>
<td>$0.910, 0.823, 0.839$</td>
</tr>
<tr>
<td>3</td>
<td>$\varepsilon_2-T_2$</td>
<td>$-0.997$</td>
</tr>
<tr>
<td>4</td>
<td>$P_1-P_2, P_1-P_3, P_1-P_4, P_2-P_3, P_2-P_4, P_2-\varepsilon_2$</td>
<td>$-0.976, 0.933, -0.889, -0.987, 0.963, -0.992$</td>
</tr>
<tr>
<td>5</td>
<td>$P_1-P_2, P_1-P_3, P_1-P_4, P_1-\varepsilon_2, P_2-P_3, P_2-P_4, P_2-\varepsilon_2, P_3-P_4, P_3-P_2, P_4-\varepsilon_2$</td>
<td>$-0.999, 0.997, -0.993, 0.985, -0.999, 0.995,-0.998, -0.998, 0.983, -0.972$</td>
</tr>
</tbody>
</table>
coefficients exceed 0.99, which means that the gradient-based method might have some difficulty in simultaneously determining relevant parameters.

3.2. Comparison of regularization techniques using the error-free measured data

Firstly, we have compared the inverse methods using the measured incident radiative heat fluxes that have no measurement error. The error-free measured data are obtained by solving the direct problem with exact boundary conditions. The number of measurement points is 48. The results obtained by various inverse methods are compared and listed in Table 3.

Since the HGA is stochastic and evaluation of the fitness value is carried out as many times as the population size, a long computational time over 10,000 s is required to satisfy the stopping criterion for cases 1–3. For cases 4 and 5, HGA cannot find an optimal value within a given time limit.

The CGM, for cases 1 and 2, accurately estimates the optimal value in a much smaller time than HGA. Furthermore, depending on the initial guess, different computational times as well as iteration numbers are required. For case 3, the same procedure is repeated with initial guesses of 0.1 up to 0.9, but CGM cannot find optimal values except 0.9. This is because parameters are highly correlated. When the correlation coefficient is even highly correlated like in cases 4 and 5, we cannot guess any proper initial guess for CGM to satisfy the stopping criterion within the time limit.

The FNM estimates parameters in a much smaller computational time and iteration number than CGM with the same initial guess for cases 1–3. For cases 3 and 4, FNM finds the optimal value even with the initial guess, whereas CGM cannot find the optimal one. However, for case 5, the FNM also cannot find an optimal value either.

In summary, as the sensitivity coefficient becomes highly correlated, it is more difficult to find a proper initial guess for successful parameter estimation, or even if it converges, many iteration numbers are required.

To overcome these difficulties, a new computational procedure is proposed as follows: as an initial guess selector, the HGA is employed and then CGM or FNM is adopted. Therefore, for the combined method of HGA + CGM or HGA + FNM, HGA is expected to provide a much better initial guess, and thereby it leads to reducing the dependence of CGM and FNM on its initial guess even in highly correlated parameter estimation problems. The final results are listed in Table 4.

Even though the iteration number is still dependent upon the initial guess provided by HGA, the computational time is observed to be much more reduced, compared with that for HGA alone. The deviation in the results obtained by the combined method also becomes smaller than that for CGM or FNM alone. Moreover, for cases 4 and 5, HGA is found to provide a good initial guess for CGM or FNM to accurately estimate results, which is hard for CGM or FNM alone to find. However, HGA + CGM still needs more computational time for cases 4 and 5 than HGA + FNM; thus the latter combined method seems to be much better for further inverse analysis.

Table 3
Comparison of iteration number and computational time for various inverse methods of HGA, CGM, and FNM

<table>
<thead>
<tr>
<th>Case</th>
<th>HGA</th>
<th>CGM</th>
<th>FNM</th>
<th>CPU time (s)</th>
<th>Initial guess Iteration no.</th>
<th>CPU time (s)</th>
<th>Initial guess Iteration no.</th>
<th>CPU time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11464</td>
<td>0.1</td>
<td>11</td>
<td>513</td>
<td>0.1</td>
<td>5</td>
<td>158</td>
<td>18681</td>
</tr>
<tr>
<td>2</td>
<td>18681</td>
<td>0.5</td>
<td>26</td>
<td>1100</td>
<td>1.0</td>
<td>6</td>
<td>213</td>
<td>16570</td>
</tr>
<tr>
<td>3</td>
<td>16570</td>
<td>0.1–0.7</td>
<td>10</td>
<td>268</td>
<td>0.9</td>
<td>5</td>
<td>138</td>
<td>–</td>
</tr>
<tr>
<td>4</td>
<td>–</td>
<td>0.1–0.9</td>
<td>–</td>
<td>–</td>
<td>0.5</td>
<td>17</td>
<td>991</td>
<td>–</td>
</tr>
<tr>
<td>5</td>
<td>–</td>
<td>0.1–0.9</td>
<td>–</td>
<td>–</td>
<td>0.9</td>
<td>5</td>
<td>138</td>
<td>–</td>
</tr>
</tbody>
</table>
3.3. Effects of measurement error and number of measurement points on estimation accuracy

With the combined method of HGA + FNM of which performance is verified above, further inverse analysis is carried out below with the conditions for case 5. Usually, measured data intrinsically contain some errors so that their effects on estimation accuracy have to be examined. In order to artificially simulate the measurement data that include some errors in itself, the following relation is used:

$$q_{\text{error},i} = q_{\text{exact},i} + \sigma_{\text{st}} \zeta_i, \quad i = 1, 2, \ldots, nm,$$

where $\sigma_{\text{st}}$ is a standard deviation of measurement data, $\zeta$ is a standard normal distribution random variable, and $nm$ is the number of measurement points. Fig. 9 shows the simulated incident radiative heat flux data that have two kinds of measurement error such as $\sigma_{\text{st}} = 500$ and 1000, which correspond to the average relative measurement errors of 2.4% and 4.79%, respectively.

<table>
<thead>
<tr>
<th>Case</th>
<th>HGA + CGM</th>
<th>HGA + FNM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Iteration no.</td>
<td>CPU time (s)</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>397</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>590</td>
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<td>602</td>
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<td>3</td>
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<tr>
<td>5</td>
<td>3232</td>
<td>161258</td>
</tr>
</tbody>
</table>

Fig. 9. Simulated incident radiative heat flux data for various error levels of $\sigma_{\text{st}} = 500$ and 1000.
For error analysis, the relative estimation error of any given parameter and the averaged relative estimation error of coefficients of polynomial are defined as follows:

Relative error: $$\phi_{r.e.} = \left| \frac{\phi_{\text{estimated}} - \phi_{\text{exact}}}{\phi_{\text{exact}}} \right| \times 100, \quad \phi = P_1, P_2, P_3, P_4, \varepsilon_2. \quad (24)$$

Averaged relative error: $$P_{a.e.} = \frac{\sum_{i=1}^{4} P_{i.r.e.}}{4}. \quad (25)$$

As shown in Table 5, without measurement errors, the emissivity and the coefficients of polynomial on surface 2 are accurately estimated with negligible relative error. When $$\sigma_{st}$$ is increased from 500 to 1000, however, the averaged relative error increases from 2.93 to 5.24, while the relative error of emissivity increases from 9.03 to 17.695. This result is also consistent with the fact that the sensitivity coefficient for emissivity is smaller than that for parameters for temperature as shown in Fig. 8(b). Based on this, it is very obvious that emissivity is more difficult to predict than parameters for temperature. The iteration number required for obtaining the solution is only slightly changed from 9 to 11. Fig. 10 illustrates that the estimated boundary temperature distribution is acceptable even for measurement errors of $$\sigma_{st} = 500$$ and 1000.

Next, the effect of measurement points on estimation accuracy is to be investigated. When measured data without measurement error are used, FNM accurately estimates parameters, even though the number of
measurement points decreases by more than 8. The iteration number required for convergence is just about 7–8. However, when the number of measurement points is decreased to less than the number of parameters, the estimation of parameters is found to diverge, because for Newton’s method the number of equations should be equal to or more than that of unknown parameters. Furthermore, when a measurement error of $\sigma_{st} = 500$ is added to exact data, the result is compared in Table 6. As the measurement points decrease, the relative error is observed to rapidly increase. Therefore, a larger number of measurement data is required for better estimation, and, if possible, the measurement data with small error or large measurement data are quite indispensable for inverse analysis.

4. Conclusions

Based on inverse radiation boundary analysis in axisymmetric absorbing, emitting, and scattering medium, various inverse methods have been compared by applying with a highly correlated case between parameters, when the measured incident radiative heat fluxes are given. The effects of measurement error and number of measurement points on estimation accuracy have also been examined.

The results found that when the parameters were highly correlated, parameter estimation using gradient-based methods tended to easily diverge. Even if it converged, the methods needed more iteration numbers to get a convergent solution. In other words, an initial guess in the vicinity of the solution was necessary to reduce the iteration number. Based on this fact, the hybrid genetic algorithm was recommended as an initial guess selector, and then the computational efficiency of the gradient-based methods was observed to be enhanced.

When there was no measurement error in the data, the Newton method showed faster convergence than the CGM for various initial guesses, even when the number of measurement points decreased to the number of unknown parameters. When there was some measurement error in the data, the relative error rapidly increased as the measurement points decreased. Finally, in the simultaneous parameter estimation of emissivity and temperature, emissivity was found to be less sensitive than temperature so that the estimation of temperature was much easier than emissivity.

Acknowledgment

The present work was supported by the Combustion Engineering Research Center at the Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology, which is funded by the Korea Science and Engineering Foundation.

References