INVERSE SURFACE RADIATION ANALYSIS
IN AN AXISYMMETRIC CYLINDRICAL ENCLOSURE
USING A HYBRID GENETIC ALGORITHM

Ki Wan Kim and Seung Wook Baek
Division of Aerospace Engineering, Department of Mechanical Engineering,
Korea Advanced Institute of Science and Technology,
Yusung-Gu, Taejon, Korea

An inverse boundary analysis of surface radiation in an axisymmetric cylindrical enclosure
is conducted in this study. The net energy exchange method was used to calculate the
radiative heat flux on each surface, and a hybrid genetic algorithm was adopted to minimize
an objective function, which is expressed by the sum of square errors between estimated and
measured or desired heat fluxes on the design surface. We have examined the effects on the
estimation accuracy of the measurement error as well as the number of measurement points.
Furthermore, the effect of a variation in one boundary condition on the other boundary
conditions is also investigated to get the same desired heat flux and temperature distribution
on the design surface.

INTRODUCTION

In various engineering applications such as furnaces, gas turbines, converging
and diverging nozzles in propulsive systems, or combustion chambers, radiation is a
major heat transfer mode in high-temperature environments. For practical analysis
of these systems, an axisymmetric assumption is often used not only to simplify the
problem, but also to reduce the computational cost [1].

When the medium inside the enclosure is a nonparticipating gas or a vacuum,
only the surface radiation needs to be taken into consideration. In this case, the net
energy exchange method, making use of view factors, is usually adopted for its
analysis. In the 1960s, study of the surface radiation in axisymmetric enclosures was
rigorously carried out for its practical concern in the design of black bodies [2, 3].

On the other hand, inverse analysis has been applied for various applications,
from diagnostic and identification problems to optimal designing and controlling
problems [4]. Inverse analysis of surface radiation in rectangular and irregular
geometries has also been conducted by Howell’s group [5, 6]. They compared various regularization techniques, and applied them to inverse design problems for radiation-dominant environments. As of now, noniterative regularization techniques such as the Tikhonov regularization method or iterative regularization techniques such as conjugate gradient or the steepest descending method, which is one of the gradient-based methods, are usually adopted to minimize the objective function [4].

However, recently, search-based methods have received much attention for their outstanding characteristics, for instance, less dependence on initial value and no need for gradient information, especially in nonlinear or multiparameter problems. Among others, a genetic algorithm (GA) has been applied to various optimization and parameter estimation problems [7]. Application of GAs to inverse radiation analysis was conducted by Li and Yang for estimating the scattering albedo, optical thickness, and phase function in parallel planes [8], while Kim et al. improved the GA by combining a local optimization technique for estimating wall emissivities in two-dimensional irregular geometry [9].

In this study, inverse boundary analysis of diagnostic as well as design problems has been applied to a cylindrical enclosure. To overcome the ill-posed character of inverse problems, a hybrid genetic algorithm as an iterative regularization technique was adopted. We have investigated the effects of the measurement error and the number of measurement points on the estimation accuracy, and the effect of variation of one boundary condition on the other boundary conditions to get the desired heat flux and temperature on the design surface.

ANALYSIS

Direct Problem

Figure 1 shows an axisymmetric cylindrical enclosure with gray, diffusely emitting, and reflecting boundaries. It is divided into a finite number of elements. Each surface has its own emissivity and temperature. The net energy exchange method is adopted to calculate the radiosity from each element on surface.

The net energy exchange on each surface can be obtained from the following equations, with nondimensionalized variables given by

\[ \zeta = \frac{x}{D}, \quad \eta = \frac{y}{D}, \quad l = \frac{L}{D}, \quad r_i' = \frac{r_i}{R} \quad i = 1, 3 \]
On surface 1,\

\[ J_1(r') = \varepsilon_1(r') \sigma T_1^4(r') + \rho_1(r') H_1(r') \] \hspace{1cm} (1a)\

\[ H_1(r') = \int_{\eta=0}^{l} J_2(\eta) \, dF_{r',-d\xi} + \int_{r''=0}^{1} J_3(r'') \, dF_{r'\rightarrow r''} \] \hspace{1cm} (1b)\

where \( J \) and \( H \) are radiosity and irradiation, \( \varepsilon \) is emissivity, while \( \rho = 1 - \varepsilon \) is reflectivity. Here, \( dF_{r',-d\xi} \) and \( dF_{r'\rightarrow r''} \) are view factors from an element on surface 1 to an element on surfaces 2 and 3, respectively, and defined as follows [2, 3]:\

\[ dF_{r',-d\xi} = \frac{8\zeta(4\zeta^2 + r'^2 - 1)}{[(4\zeta^2 + r'^2 + 1)^2 - 4r'^2]^{3/2}} \, d\zeta \] \hspace{1cm} (1c)\

\[ dF_{r'\rightarrow r''} = \frac{1}{r'^2} \left\{ \frac{2R' \ell^2 (R^2 + R'^2 + 1)}{[(R^2 + R'^2 + 1)^2 - 4R'^2]^{3/2}} \right\} \, dr'' \] \hspace{1cm} (1d)\

where \( R' = r''/r' \) and \( \ell = 2l/r' \).

In a similar way, on surface 2,\

\[ J_2(\zeta) = \varepsilon_2(\zeta) \sigma T_2^4(\zeta) + \rho_2(\zeta) H_2(\zeta) \] \hspace{1cm} (2a)\

\[ H_2(\zeta) = \int_{\eta=0}^{l} J_2(\eta) \, dF_{\xi,-d\eta}(|\eta - \zeta|) \]

\[ + \int_{r''=0}^{1} J_1(r'') \, dF_{\xi\rightarrow r''} + \int_{r''=0}^{1} J_3(r'') \, dF_{\xi\rightarrow r''} \] \hspace{1cm} (2b)
\[ dF_{d \xi - d \eta}(\eta - \xi) = \left\{ 1 - \frac{|\eta - \xi|}{2[(|\eta - \xi|)^2 + 1]^{3/2}} \right\} d\eta \tag{2c} \]

\[ dF_{d \xi - r'_i} = \frac{4\zeta(4\zeta^2 + r_i^2 - 1)}{[(4\zeta^2 + r_i^2 + 1)^3 - 4r_i^2]^{3/2}} d\xi \] \[ i = 1, 3 \tag{2d} \]

and on surface 3,

\[ J_3(r'_3) = e_3(r'_3) \sigma T_3^4(r'_3) + \rho_3(r'_3)H_3(r'_3) \tag{3a} \]

\[ H_3(r'_3) = \int_{\eta = 0}^{l} J_2(\eta)dF_{r'_3 - d\xi} + \int_{r'_1 = 0}^{l} J_1(r'_1)dF_{r'_3 - r'_1} \tag{3b} \]

\[ dF_{r'_3 - d\xi} = \frac{8\zeta(4\zeta^2 + r_3^2 - 1)}{[(4\zeta^2 + r_3^2 + 1)^3 - 4r_3^2]^{3/2}} d\xi \tag{3c} \]

\[ dF_{r'_3 - r'_i} = \frac{1}{r'_3} \left\{ \frac{2R'l^2(R^2 + 1)}{[(l^2 + R^2 + 1)^3 - 4R^2]^{3/2}} \right\} dr'_i \tag{3d} \]

where \( R' = r'_i/r'_3 \) and \( l' = 2l/r'_3 \).

To accurately calculate the above equations, which are well known as the Fredholm integral equation of the second kind, the Nyström method was adopted [10]. Heat flux on each surface element can be obtained with radiosity calculated from Eqs. (1)–(3) as follows:

\[ q''(\zeta) = \frac{e(\zeta)}{1 - e(\zeta)} [E_b(\zeta) - J(\zeta)] \tag{4} \]

where \( E_b(\zeta) = \sigma T_3^4(\zeta) \) is the blackbody emissive power, and \( \sigma = 5.670 \times 10^{-8} \) W/\( m^2K^4 \) is the Stefan-Boltzmann constant.

**Hybrid Genetic Algorithm**

The genetic algorithm (GA), based on the concept of the natural selection, is a robust parameter estimator. It represents and manipulates candidate solutions at the genotype, and each candidate solution’s group is called an individual. Individuals randomly generated at the first stage reach the global optimum through main operators such as selection, crossover, and mutation as generation number increases, thus it has no influence of initial values.

Figure 2 shows a flowchart for the hybrid genetic algorithm (HGA) used in this study. At the first initialization stage, an initial population, which is a set of individuals, is randomly generated within the design space. Each individual forms an array of candidate solutions which are represented in binary or float-point. In this study, the population size is fixed to 10 to reduce the computational time, which is a
The main drawback of the GA, and float-point representation is used to reduce the length of the chromosome to the number of design variables.

The fitness of each individual must be evaluated to determine which individuals survive to the next generation and which individuals end life in this generation. The evaluation of fitness is done by calculating the objective function with each individual’s candidate solutions. This operation requires most of the computational time used in the genetic operation. Since the evaluation is carried out by fitness value, gradient information is not needed for optimization.

Through the selection operation, a fitter individual among the individuals of the present generation is selected to reproduce offspring for the next generation. There are various selection schemes, such as fitness-proportional selection, ranking selection, and tournament selection. In this study, fitness-proportional selection is adopted, and thus the fitter individual with higher probability is to be selected. However, when the population size is small, individuals of the next generation are likely to be filled with superindividuals that are superior to other individuals, so that
the individuals fall into the local optimum in the end. To prevent this and keep the diversity of individuals, stochastic universal sampling is used for selection operator.

The individuals chosen by the selection operation undergo the operations of crossover and mutation for reproduction of fitter individuals. In the crossover operation, individuals, which are as many as the number corresponding to the probability of crossover, meet their mates and exchange their genes. In this study, one-point crossover is used, and thus two individuals swap their genes at one crossing point. The mutation operator allows some genes to change their value within the design space to find fitter individuals, and the probability of mutation controls the number of genes to undergo the mutation operation. Nonuniform mutation is adopted to improve the fine local tuning ability as the generation number increases. An elite strategy is used to ensure a monotonic improvement by copying the best individual of the present generation to the next generation.

A local optimization algorithm (LOA) is often included in the GA in order to overcome such disadvantages as the inability of fine local tuning, so this type of GA is called a hybrid genetic algorithm (HGA). The LOA makes the individuals move from near optimal range to optimal point more quickly. In this study, an operator used for nonuniform mutation is adopted for the LOA. Usually, the gradient-based optimization technique is used for the LOA. However, to keep the stochastic feature of the GA, a search-based technique is adopted. After determining the elite individual in the elite strategy, the LOA is applied only to elite individuals, to reduce computational time as well as to keep the diversity of individuals.

If \( s = (v_1, v_2, \ldots, v_m) \) is a chromosome of an elite individual and gene \( v_k \) is selected for local optimization, the resulting gene \( v'_k \) is obtained as follows [7]:

\[
v'_k = \begin{cases} 
  v_k + \Delta(t, UB - v_k) \\
  v_k - \Delta(t, v_k - LB)
\end{cases}
\]

where LB and UB are lower and upper domain bounds for the gene \( v_k \).

The function \( \Delta(t, y) \) returns a value in the range \([0, y]\) such that the probability of \( \Delta(t, y) \) being close to 0 increases as \( t \) increases. The following function is used for \( \Delta(t, y) \):

\[
\Delta(t, y) = y \cdot \left(1 - r^{(1-t/T_{\text{max}})^b}\right)
\]

where \( r \) is a uniform random number, \( T_{\text{max}} \) is the maximum generation number, and \( b \) is a system parameter for determining the degree of dependency on generation number, \( t \) (\( b = 1 \) here).

In the LOA, \( v'_k \) is calculated for each gene of the elite individual using Eq. (5). If \( v'_k \) fits better than \( v_k \), the gene of the elite individual is changed to \( v'_k \). Otherwise, \( v_k \) is maintained. The detailed behavior of the HGA used in this study is found in the literature [9].

**Inverse Analysis Procedure**

This inverse radiation analysis in this study consists of diagnostic and design problems. First, unknown parameters in diagnostic problem are to be estimated by
minimization of an objective function which is expressed by the sum of square errors between estimated and measured heat fluxes at measurement positions on the design surface, while assuming the other physical values are all known.

In the design problem, boundary values on one side, which satisfy the desired heat flux and temperature distribution on the design surface, are to be estimated when a boundary condition on the other side is changed. This can be achieved by minimizing the objective function formed by the sum of square errors between the estimated and desired heat fluxes while the temperature on the design surface is fixed to the desired temperature.

The objective function for minimization in this case is defined by

\[ f = \sum_{i=1}^{N} (q_{i,A}^* - q_{i,B})^2 \]  

where \( N \) is the number of measurement points and \( q^* \) is the nondimensional heat flux. The subscript \( A \) indicates an estimated value, \( B \) represents a measured value for the diagnostic problem and a desired value for design problem. As mentioned previously, the hybrid genetic algorithm is be adopted for minimization of the objective function, Eq. (7).

RESULTS AND DISCUSSION

Direct Problem

First of all, in order to determine the adequate number of elements on the design surface, the heat flux distribution was obtained on surface 2 with boundary conditions \( T_1 = T_3 = T_{\text{ref}}/2, \ T_2 = T_{\text{ref}}, \) and \( \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.9. \) Figures 3a and 3b show the distribution of radiosity and heat flux, respectively, on surface 2. Here, temperature, heat flux, and radiosity are nondimensionalized as follows: \( T_i^* = T_i/T_{\text{ref}}, \ q_i^* = q_i^0/\sigma T_{\text{ref}}^4, \) and \( J_i^* = J_i/\sigma T_{\text{ref}}^4. \) The result shows that the distributions do not change when more than 70 elements are used on surface 2. The element number used on surfaces 1 and 3 is 20 for each surface.

Inverse Diagnostic Analysis

Case 1. Simultaneous estimation of \( T_1^* \) and \( T_3^* \). Now, nondimensional side-wall temperatures, i.e., \( T_1^* \) and \( T_3^* \), are to be estimated when measured heat fluxes are available on surface 2. Since the local optimization method used in this study is a function of random number, the effect of random number seed on the performance of the HGA is investigated. As shown in Figures 4a and 4b, there are no significant differences in the best fit for various random number seeds, and the desired side-wall temperatures of 0.5 is accurately estimated regardless of random number seed. Based on these results, the random number seed is fixed in forthcoming analyses.

Case 2. Simultaneous estimation of \( T_1^*, \ T_3^*, \ \varepsilon_1, \) and \( \varepsilon_3 \). In this case, both nondimensional temperature and emissivity on each side wall are to be simultaneously estimated when heat flux data measured on surface 2 are available. Table 1
shows that the estimated values of wall temperatures and emissivities have relative errors of about 4% and 1%, respectively, which are obtained even with measured data without errors. Still, the HGA yields a relatively accurate estimation. These results are due to the fact that because the sensitivity coefficients of temperature

Figure 3. (a) Nondimensional heat flux. (b) Radiosity on surface 2 for various numbers of elements.
Figure 4. (a) Variation of best fit with generation. (b) Variation of nondimensional temperature on surface 2 for various random number seeds.
and emissivity on the same side wall are closely dependent each other, a simultaneous prediction of two values would be very difficult. Additionally, the relative error for temperature is observed to be greater than that for emissivity, since the sensitivity coefficient for emissivity, which is a measure of the effects of estimated value on measured data, is greater than that for temperature, as shown in Figure 5. Sensitivity coefficients of parameter $\phi_i$ can be estimated using finite-difference approximation as follows [4]:

![Figure 5. Sensitivity coefficients for wall temperature and emissivity for case 2.](image)

<table>
<thead>
<tr>
<th>Range</th>
<th>True value</th>
<th>Estimated value</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_1$</td>
<td>0–1</td>
<td>0.9</td>
<td>0.9125</td>
</tr>
<tr>
<td>$\varepsilon_3$</td>
<td>0–1</td>
<td>0.9</td>
<td>0.8907</td>
</tr>
<tr>
<td>$T_1^\ast$</td>
<td>0–1</td>
<td>0.5</td>
<td>0.5215</td>
</tr>
<tr>
<td>$T_3^\ast$</td>
<td>0–1</td>
<td>0.5</td>
<td>0.4798</td>
</tr>
</tbody>
</table>
\[
Z_{ij} = \frac{\partial q^*_2}{\partial \phi_i} \approx \frac{q^*_2(\phi_1, \ldots, \phi_i + \varepsilon \phi_i, \ldots, \phi_I) - q^*_2(\phi_1, \ldots, \phi_i, \ldots, \phi_I)}{\varepsilon \phi_i}
\]

for \( i = 1, \ldots, I \) and \( j = 1, \ldots, N \)

where \( I \) is the number of parameters to be estimated (here \( I = 4 \)), \( j \) is the number of measurement points, and \( \varepsilon \) is a very small number.

**Case 3. Estimation of emissivities on surface 2 with 10 elements.** In this test case, the surface 2 is subdivided into 10 elements for multiparameter estimation using the HGA, while considering the effect of measurement error on the accuracy of emissivity estimation. Since the gradient information, i.e., the sensitivity coefficient, is not necessary for the genetic algorithm, it is extremely useful, especially for the case of a multiparameter problem such as the current problem. To simulate experimental data containing some measurement errors, the following relation is used:

\[
(q^*_2)_{\text{measured}} = (q^*_2)_{\text{exact}} + \sigma_{st} \zeta \quad i = 1, 2, \ldots, N
\]

where \( \sigma_{st} \) is the standard deviation of the measurement data, and \( \zeta \) is a standard normal distribution random variable.

For error analysis, relative and averaged relative errors are defined by

\[
\text{Relative error : } \varepsilon_{2,i} = \left| \frac{\varepsilon_{\text{estimated},i} - \varepsilon_{\text{exact},i}}{\varepsilon_{\text{exact},i}} \right| \times 100 \quad i = 1, 2, \ldots, 10
\]

\[
\text{Averaged relative error : } \varepsilon_2 = \frac{\sum_{i=1}^{10} \varepsilon_{2,i}}{10}
\]

As shown in Table 2, without measurement errors, the emissivities on surface 2 are quite accurately estimated. When \( \sigma_{st} \) increases from 0.001 to 0.01, however, the averaged relative error increases from 0.78 to 4.08. Here, \( \sigma_{st} \) of 0.001 and 0.01 correspond to average measurement errors of 1.25\% and 12.5\%, respectively. For the same standard deviation, the relative error becomes largest at a certain element, which is, in our thought, due to the stochastic feature of the HGA. Nevertheless, it is found that the hybrid genetic algorithm successfully estimates the multielemental emissivities in this multiparameter case.

**Case 4. Estimation of \( T_1^* \), \( T_3^* \), and \( \varepsilon_2 \).** In this case, the simultaneous estimation of \( T_1^* \), \( T_3^* \), and \( \varepsilon_2 \) is to be carried out, and the effect of the number of measurement points on the solution accuracy is also examined. In sensitivity analysis, the sensitivity coefficients for temperature and emissivity vary independently of each other as shown in Figure 6, so simultaneous estimation of both values is feasible. Even though the number of measurement points decreased more than by half, a change in relative errors is insignificant as shown in Table 3. This result represents that the effect of number of measurement points on the solution accuracy is negligible, which might be due to a mutual independency of two sensitivity coefficients over the measurement range. Here, it is again reconfirmed that the independency of two sensitivity coefficients is crucial to the simultaneous estimation of two conditions.
Table 2. Emissivities estimated on surface 2 for different standard deviations

<table>
<thead>
<tr>
<th>Point (10 ea)</th>
<th>Range</th>
<th>True value</th>
<th>Estimated value</th>
<th>Relative error (%)</th>
<th>Estimated value</th>
<th>Relative error (%)</th>
<th>Estimated value</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{2,1}$</td>
<td>0–1</td>
<td>0.9</td>
<td>0.90019</td>
<td>0.02</td>
<td>0.90091</td>
<td>0.10</td>
<td>0.89871</td>
<td>0.37</td>
</tr>
<tr>
<td>$\varepsilon_{2,2}$</td>
<td>0–1</td>
<td>0.9</td>
<td>0.90140</td>
<td>0.15</td>
<td>0.90330</td>
<td>0.36</td>
<td>0.91436</td>
<td>1.58</td>
</tr>
<tr>
<td>$\varepsilon_{2,3}$</td>
<td>0–1</td>
<td>0.9</td>
<td>0.90351</td>
<td>0.39</td>
<td>0.90256</td>
<td>0.28</td>
<td>0.91823</td>
<td>2.02</td>
</tr>
<tr>
<td>$\varepsilon_{2,4}$</td>
<td>0–1</td>
<td>0.9</td>
<td>0.90688</td>
<td>0.76</td>
<td>0.90795</td>
<td>0.88</td>
<td>0.97076</td>
<td>7.86</td>
</tr>
<tr>
<td>$\varepsilon_{2,5}$</td>
<td>0–1</td>
<td>0.9</td>
<td>0.90877</td>
<td>0.97</td>
<td>0.89016</td>
<td>1.09</td>
<td>0.78581</td>
<td>12.7</td>
</tr>
<tr>
<td>$\varepsilon_{2,6}$</td>
<td>0–1</td>
<td>0.9</td>
<td>0.90728</td>
<td>0.80</td>
<td>0.90598</td>
<td>0.66</td>
<td>0.90941</td>
<td>1.03</td>
</tr>
<tr>
<td>$\varepsilon_{2,7}$</td>
<td>0–1</td>
<td>0.9</td>
<td>0.90405</td>
<td>0.45</td>
<td>0.93667</td>
<td>4.07</td>
<td>1.00000</td>
<td>11.1</td>
</tr>
<tr>
<td>$\varepsilon_{2,8}$</td>
<td>0–1</td>
<td>0.9</td>
<td>0.90179</td>
<td>0.19</td>
<td>0.90185</td>
<td>0.20</td>
<td>0.91053</td>
<td>1.16</td>
</tr>
<tr>
<td>$\varepsilon_{2,9}$</td>
<td>0–1</td>
<td>0.9</td>
<td>0.90050</td>
<td>0.05</td>
<td>0.90209</td>
<td>0.23</td>
<td>0.92633</td>
<td>2.92</td>
</tr>
<tr>
<td>$\varepsilon_{2,10}$</td>
<td>0–1</td>
<td>0.9</td>
<td>0.90014</td>
<td>0.01</td>
<td>0.90010</td>
<td>0.01</td>
<td>0.90070</td>
<td>0.06</td>
</tr>
</tbody>
</table>

$\varepsilon_2$ | 0.37 | 0.78 | 4.08 |

Figure 6. Sensitivity coefficients for wall temperature and emissivity for case 4.
Inverse Boundary Condition Design

Case 1. The Effect of $e_2$ on $T_1^*$ and $T_3^*$. In this case, when $e_2$ is changed from its original value of 0.9 to 0.81, a change in $T_1^*$ and $T_3^*$ is to be examined, while still satisfying the desired temperature and heat flux on surface 2 for the direct problem with $e_2 = 0.9$. Figure 7 illustrates that as the emissivity $e_2$ on surface 2 decreases from the original value of 0.9, $T_1^*$ and $T_3^*$ should decrease to hold constant the heat flux emitted from surface 2, but the errors between the desired and estimated heat flux are observed to increase. In other words, the irradiation from surfaces 1 and 3 should get smaller by lowering its own temperature, since the emissivity on surface 2 decreases.

Table 3. Temperatures and emissivities estimated for different numbers of measurement points

<table>
<thead>
<tr>
<th>Range</th>
<th>$e_2$</th>
<th>$T_1^*$</th>
<th>$T_3^*$</th>
<th>$N$</th>
<th>$N/2$</th>
<th>$N/4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>True value</td>
<td>Estimated value</td>
<td>Relative error (%)</td>
<td>Estimated value</td>
<td>Relative error (%)</td>
<td>Estimated value</td>
</tr>
<tr>
<td>0–1</td>
<td>0.9</td>
<td>0.89955</td>
<td>0.05</td>
<td>0.89768</td>
<td>0.25</td>
<td>0.89907</td>
</tr>
<tr>
<td>0–1</td>
<td>0.5</td>
<td>0.49688</td>
<td>0.62</td>
<td>0.49556</td>
<td>0.88</td>
<td>0.49946</td>
</tr>
<tr>
<td>0–1</td>
<td>0.5</td>
<td>0.49755</td>
<td>0.49</td>
<td>0.49557</td>
<td>0.88</td>
<td>0.49840</td>
</tr>
</tbody>
</table>

Figure 7. Variation of error and temperatures of surfaces 1 and 3 against the emissivity on surface 2.
while maintaining the temperature of surface 2 at the same desired temperature. It is also observed that a small change in emissivity on surface 2 incurs a relatively large change in temperatures, since the sensitivity coefficient of the heat flux with respect to emissivity is high, as mentioned before.

**Case 2. The Effect of $T_1^*$ on $T_3^*$.** In this case, when $T_1^*$ is changed from its original value of 0.5 to 1.0, a change in $T_3^*$ is to be examined, while still satisfying the desired temperature and heat flux on surface 2 for the direct problem with $\varepsilon_2 = 0.9$. As shown in Figure 8, as $T_1^*$ increases from 0.5, the errors between the desired and estimated heat flux on surface 2 become larger, and the $T_3^*$ becomes smaller to accommodate a change in $T_1^*$ as well as to sustain the imposed boundary condition on surface 2. In this case, however, a large change in $T_1^*$ entails only a small change in $T_3^*$, which can be attributed to the fact that the sensitivity coefficient of temperature of surface 1 against the heat flux on surface 2 is small.

**CONCLUSIONS**

Based on the inverse boundary analysis of surface radiation in an axisymmetric cylindrical enclosure, the effect of the measurement error and the number of
measurement points on the estimation accuracy has been examined. Also, the effect of the variation of one boundary condition on the others was investigated for the same desired heat flux and temperature on the design surface.

The results showed that the hybrid genetic algorithm accurately estimated unknown parameters in a multiparameter problem, even for the case with some measurement errors. The effect of the number of measurement points on the estimation accuracy was observed to be negligible when the sensitivity coefficients for two variables varied independently, especially for the case of a simultaneous estimation problem. In addition, the inverse analysis with the HGA predicted very well the effect of a change in one boundary condition on the other boundary conditions, while satisfying the desired boundary conditions such as heat flux or temperature. Finally, the sensitivity analysis provided useful information for simultaneous parameter estimation as well as for prediction of the relative variation in boundary condition for application to inverse design. Even if the unknown parameters are functions of position, there is no difficulty in solving them with the present methodology. Just the number of parameters to be estimated would be increased.

REFERENCES