Heat Transfer to a Composite Material Under Ice Particle Impacts

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This paper examines what would happen if a supersonic vehicle flies through an atmosphere laden with ice particles. In previous research of the present authors, an experiment was performed to determine the characteristics of ice-particle impact phenomenon. Therein, the mass loss from the vehicle’s surface material by the impacts was measured and the fragments’ behavior was studied. In the present work, the trajectories of fragments from the target material mass removed by impact are explored. The results of such studies have not yet been reported in whole, at least in open literature.

When a high-speed vehicle flies through a cloud of rain drops, snowflakes, or hail, damage may occur to the vehicle. Although hail is rare, rain drops exist as ice particles at high altitudes [1-3]. The impacts of such ice particles on a high-speed vehicle are a concern worthy of study. An environment in which solid or liquid particles impact a flying object is known as an erosive environment. The impacts increase the wearing rate of the surface material by two mechanisms: direct gouging of material by forming impact craters and increase in thermochemical ablation rate by an increase in heat transfer rates. Some amount of research has been carried out on this subject [4,5]. However, the results of such studies have not yet been reported in whole, at least in open literature.

The phenomenon of impact of a solid object with a solid surface at a high speed has been studied and reported in open literature in

I. Introduction

Nomenclature

- \( c_1, c_2, c_3 \) = nondimensional coefficient for fragments’ mass distribution, Eq. (3)
- \( c_5 \) = nondimensional coefficient for fragments’ velocity distribution, Eq. (6)
- \( D \) = fragments’ diameter
- \( E \) = projectile kinetic energy, J
- \( F \) = distribution function, Eq. (6)
- \( f \) = stream function, Eq. (11)
- \( H \) = flow enthalpy, J/kg
- \( H_t \) = heat of conversion of 1 g of ice at 0°C to steam at 100°C 3012 J/kg
- \( m \) = mass of a particle
- \( q \) = stagnation-point heat transfer rate without ice impact
- \( q_i \) = stagnation-point heat transfer rate with ice impact
- \( q_{st} \) = stagnation-point heat flux
- \( R \) = nose radius
- \( Re \) = Reynolds number
- \( S_t \) = Stanton number
- \( T \) = temperature, K
- \( t \) = time
- \( u \) = velocity in x direction
- \( V \) = magnitude of velocity, \( \sqrt{u^2 + v^2} \)
- \( v_i \) = equivalent injection velocity
- \( v \) = velocity in y direction
- \( x \) = horizontal distance
- \( y \) = vertical distance
- \( \alpha_i \) = fraction of ice mass in the atmosphere
- \( \beta \) = ratio of the target material mass removed by impact to the mass of impacting ice particle
- \( \gamma \) = nondimensional exponent, Eq. (7)
- \( \Delta M \) = displaced mass, g
- \( \delta \) = turbulent mixing length
- \( \Phi \) = heat flux blockage factor
- \( \phi \) = inclination angle of the ejecta with respect to the normal
- \( \mu \) = viscosity, N-s/m²
- \( \nu \) = turbulent viscosity, N-s/m²
- \( \rho \) = density
- \( \sigma \) = scale parameter, Eq. (4)
- \( \tau \) = injection-induced turbulence parameter, sec, Eq. (19)
- \( \chi \) = fragment mass-to-air mass ratio, \( \alpha_i (1 + \beta) \)
- \( \omega \) = half-width angle, deg, Eq. (5)

Subscripts

- \( a \) = ambient condition
- \( D \) = size
- \( i \) = impact
- \( m \) = mass
- \( max \) = maximum value
- \( min \) = minimum value
- \( rel \) = relative value
- \( s \) = shock layer
- \( t \) = target material
- \( v \) = velocity
- \( w \) = wall
- \( \phi \) = ejecta angle


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connection with the astronomical impacts [6]. It is well known that when an asteroid impacts a planetary surface a crater is formed. In the cratering process the impacting body is fragmented and, when the impact energy is sufficient, it is vaporized. The target surface loses mass by forming a crater. The mass ejected from the surface is also fragmented and sometimes vaporized.

A theoretical model [7] to describe this cratering phenomenon should predict the quantity of the mass removed from the surface and the momentum and energy imparted to the surface. To develop a theoretical model to describe the ice-particle impact phenomenon, the quantity of the mass removed from the surface, the magnitudes of the momentum and energy carried by the issuing mass, and the direction of the issuing mass were measured experimentally [8] by the present authors. The results of this study provide a formula that could be used in determining the surface mass loss rate due to direct gouging.

The behavior of the issuing mass has an impact on the second mechanism of mass removal, i.e., increase in heat transfer rate. It has been observed that, in an erosive environment, the bow shock wave formed over a blunt body bulges forward, and shock layer pressure and heat transfer over the body increase greatly [9,10]. The exact relationship between the mass, momentum, and energy of the issuing mass and the extent of increase in heat transfer is not known presently. However, it is suspected that the increase in the heat transfer is because of the turbulence generated by the fragments [11,12].

The objective of the present work is to study the consequence of ice-particle impacts on heat transfer phenomenon. A composite material is chosen here because it is commonly used as a structural material for high-speed vehicles. A new turbulence model, a modified version of the injection-induced turbulent model [11], is used in describing this heating environment. An assumption was introduced that the turbulent mixing length is proportional to the depth of the impact craters. The constant of proportionality was determined from the existing experimental data taken in wind-tunnel tests. The model is used in determining the heat flux enhancement in the stagnation region.

II. Summary of Earlier Experimental Results

The heatshield of a reentry vehicle is made in many cases with a composite material. One of those composite materials is carbon/epoxy. In [8], the effect of ice-particle impacts on a carbon/epoxy was studied experimentally. An ice projectile was accelerated by a two-stage light gas gun. The Mach number of the projectile ranged from 2 to 3. A carbon/epoxy composite material was prepared for the target specimen. Test sets and results are summarized in Table 1.

In that experiment, a total mass removed from target specimen was determined to be expressible as

\[ \Delta M = -0.00327 + 0.000648 \times E \]  

(1)

In the above equation, \( \Delta M \) and \( E \) denote removed mass in g and projectile kinetic energy in J. From the above equation, crater size was determined as

\[ d = \frac{3}{2} \left( \frac{\Delta M}{\rho_c} \right)^{1/3} \]  

(2)

where \( d \) is the equivalent crater size. From here on, attention will be focused on the increase in heat transfer rate due to particle impacts. An impact produces fragments. The issuing fragments consist of the broken-up pieces of the ice projectile and target material in a powered form. The behavior of ice fragments is known from [13]. The behavior of target material was studied using a witness film and a high-speed camera. Figure 1 shows the issuing mass distribution for tests 08071702, 08071703, and 08071802. From Fig. 1, the mass distribution curve was derived as

\[ N(V) = c_1 c_2 \exp \left( -c_2 V / V_0 \right) + \left( 1 - c_1 c_3 \right) \exp \left( -c_3 V / V_0 \right) \]  

(3)

where the constants \( c_1 \), \( c_2 \), and \( c_3 \) are 0.2452, 26820, and 4118, respectively. \( V_0 \), a scale parameter related to the total volume, is

Table 1  Ice impact test sets for specimens

<table>
<thead>
<tr>
<th>Test number</th>
<th>Energy, J</th>
<th>Displaced mass, g</th>
</tr>
</thead>
<tbody>
<tr>
<td>08042501</td>
<td>6.812</td>
<td>N/A</td>
</tr>
<tr>
<td>08042502</td>
<td>6.516</td>
<td>N/A</td>
</tr>
<tr>
<td>08042503</td>
<td>8.224</td>
<td>N/A</td>
</tr>
<tr>
<td>08042506</td>
<td>9.593</td>
<td>N/A</td>
</tr>
<tr>
<td>08042507</td>
<td>7.647</td>
<td>N/A</td>
</tr>
<tr>
<td>08042508</td>
<td>6.107</td>
<td>0.0002</td>
</tr>
<tr>
<td>08042601</td>
<td>8.124</td>
<td>0.0018</td>
</tr>
<tr>
<td>08042602</td>
<td>8.538</td>
<td>0.0025</td>
</tr>
<tr>
<td>08042604</td>
<td>9.988</td>
<td>0.0037</td>
</tr>
<tr>
<td>08042605</td>
<td>9.342</td>
<td>0.0026</td>
</tr>
<tr>
<td>08042606</td>
<td>6.239</td>
<td>0.0001</td>
</tr>
<tr>
<td>08042607</td>
<td>6.812</td>
<td>0.0011</td>
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<td>08050901</td>
<td>6.972</td>
<td>0.0014</td>
</tr>
<tr>
<td>08050902</td>
<td>6.460</td>
<td>0.0011</td>
</tr>
<tr>
<td>08050903</td>
<td>10.313</td>
<td>0.0032</td>
</tr>
<tr>
<td>08051001</td>
<td>7.293</td>
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</tr>
<tr>
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</tr>
<tr>
<td>08051003</td>
<td>5.428</td>
<td>0.00005</td>
</tr>
<tr>
<td>08051101</td>
<td>8.602</td>
<td>0.0004</td>
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<tr>
<td>08051102</td>
<td>6.002</td>
<td>0.00085</td>
</tr>
<tr>
<td>08051103</td>
<td>9.778</td>
<td>0.003</td>
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<tr>
<td>08051202</td>
<td>6.820</td>
<td>0.0016</td>
</tr>
<tr>
<td>08062701</td>
<td>7.292</td>
<td>0.0010</td>
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<tr>
<td>08062902</td>
<td>5.762</td>
<td>0.0007</td>
</tr>
<tr>
<td>08063002</td>
<td>10.598</td>
<td>0.0024</td>
</tr>
<tr>
<td>08071702</td>
<td>6.820</td>
<td>N/A</td>
</tr>
<tr>
<td>08071703</td>
<td>6.820</td>
<td>N/A</td>
</tr>
<tr>
<td>08071802</td>
<td>8.004</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Fig. 1 Cumulated number larger than volume (V) for tests 08071702, 08071703, and 08071802.

5.33 × 10^7 µm³. Scale parameters of ice fragments \( \sigma_i \) and of target \( \sigma_t \) are 52 µm and 35.15 µm, respectively. From these values the representative characteristic length \( \sigma \) was determined as

\[
\sigma = \frac{1}{(V_{ice} + V_{target})^{1/3} + (V_{ice} + V_{target})^{1/3}} \left[ \frac{V_{ice}}{V_{ice} + V_{target}} \right]^{1/3} \sigma_i + \left[ \frac{V_{target}}{V_{ice} + V_{target}} \right]^{1/3} \sigma_t
\]  

(4)

where \( V_{ice} \) and \( V_{target} \) are volumes of fragments of ice projectile and target material, respectively.
The directional distribution of solid particles is shown in Fig. 2. Ejecta angle $\phi$ is the direction of ejecta fragments, i.e., inclination angle with respect to the normal direction. From this figure, it can be found that many solid particles are issued at high angles.

The fragments’ velocity was also empirically formulated in [8] as

$$V(D, \phi) = V_i \cdot F_{v\phi}(\phi) \cdot F_{v,D}(D)$$ (5)

where $V$ is the fragments’ velocity and $V_i$ is the projectile velocity. Here, $F_{v\phi}(\phi)$ represents the effect of ejecta angle. Figure 3 shows the fragments’ velocity divided by the projectile velocity in the ejecta angle region for several tests. The line fitted to the data is

$$F_{v\phi}(\phi) = c_s \exp\left( -\frac{(\phi - \phi_{i,\text{max}})^2}{2\sigma_i^2} \right)$$ (6)

where constant $c_s$ is 1.2152, and $\phi_{i,\text{max}}$, the angle where the maximum fragment velocity occurs, and $\sigma_i$, the half-width, are 82.12 and 11.706 deg, respectively.

In Eq. (5), $F_{v,D}(D)$ represents the effect of fragments’ size $D$

$$F_{v,D}(D) = \left( \frac{D}{D_{\text{min}}} \right)^{-\gamma}$$ (7)

where $D_{\text{min}}$ and exponent $\gamma$ are 1 $\mu$m and 0.366, respectively.

### III. Motion of Rebounding Fragments

#### A. Governing Equations

Motion of solid particles issuing from the heat shield wall was studied in [14–16]. In those studies, the issuing particles were assumed to be spherical in shape, and so the drag coefficient would be unity, which is the Newtonian value. This assumption was made here also. The density of the fragments was assumed to be that of ice, that is 0.919 g/cm$^3$.

To simplify the problem, only the stagnation region flow, sketched in Fig. 4, was considered here. The motion of an issuing fragment was dictated by the Newton’s equation of motion. The force acting on a particle is the drag. It was calculated as a vectorial sum of the force due to flow motion and force due to particle motion. The angle between the flow and the particle is described by $\theta$. The $x$- and $y$-components of the drag force produce the acceleration

$$m \frac{du}{dt} = -\text{drag} \times \cos(\theta) \quad m \frac{dv}{dt} = -\text{drag} \times \sin(\theta)$$ (8)

The particle is heated on the windward side. Total heat transfer to the particle is calculated as the product of the stagnation-point heat transfer rate and the frontal surface area $\pi D(D/2)^2$. In the calculation of the stagnation-point heat transfer rate, a simplification $\rho u = \text{constant}$, is made. Using Goulard’s analysis and approximating the parameter $\sqrt{\rho_v/\rho}$, to be $\sqrt{1/8}$, the Goulard’s formula [17] for stagnation-point heat transfer rate $q_{st}$ becomes

$$q_{st} = 0.590 \rho V_{\text{rel}}^2 (H - H_w)/\sqrt{Re}$$ (9)

where the density $\rho$ is $\rho_w$ when the particle is inside the shock layer and $\rho_v$ when it is outside of the shock layer. The Reynolds number $Re$ is

$$Re = \frac{\rho V_{\text{rel}} (D/2)}{\mu_w}$$ (10)

The flow and wall enthalpy difference $H - H_w$ is $V_{\text{rel}}^2/2 + 1006 \times (T - T_w)$ in J/kg.

Because of vaporization, the heat transfer rate is convectively blocked partly. By defining the stream function $f$ by

$$f = -\frac{\rho u}{\sqrt{2\rho_v \mu_v (du/dx)}}$$ (11)

![Fig. 4 Schematic of the flowfield of ice-impact phenomenon.](image)
the heat transfer rate to a vaporizing surface is the value given by Eq. (9) times the blockage factor \( \Phi \), which is smaller than one and is a function of the wall velocity of \( f \). The acceleration parameter \( du/dx \) is given from the Newtonian theory as

\[
\frac{du}{dx} = \frac{V_{rel}}{D/2} \sqrt{2 \rho_u / \rho_s}
\]

(12)

Numerical solution of the heat transfer problem, such as that in [16], shows that \( \Phi \) is approximately a linear function of \( f_w \) in the small range of \( f_w \), as

\[
\Phi = 1 - f_w
\]

(13)

The mass flow rate of the vapor at the stagnation point of a fragment \( \rho v \), appearing in Eq. (11), is calculated in general by dividing the heat transfer rate value \( q_a \) by the rate of vaporization \( H_v = 3012 \text{ J/kg} \)

\[
\Phi = 1 - \frac{q_a/H_v}{\sqrt{2\rho_u/\mu_s}(du/dx)}
\]

(14)

Therefore, the blockage-corrected stagnation-point heat transfer rate for a fragment \( q_a \), satisfies the equation

\[
q_a = \frac{0.590 \rho V_{rel}(H - H_w) \sqrt{Re}}{1 + 0.590(h - h_w)/H_w}
\]

(15)

By solving the above equation for \( q_a \), one obtains

\[
q_a = \frac{0.590 \rho V_{rel}(H - H_w) \sqrt{Re}}{1 + 0.590(h - h_w)/H_w}
\]

(16)

Table 2 Selected flow environment and stagnation-point heat flux rate enhancement factor calculated by crater-induced turbulence model.

<table>
<thead>
<tr>
<th>Altitude, km</th>
<th>Density, kg/m³</th>
<th>Flight velocity, m/s</th>
<th>Ice projectile mass, mg</th>
<th>( \beta )</th>
<th>( \sigma, \mu m )</th>
<th>( q_a/q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>0.746</td>
<td>500</td>
<td>18.84</td>
<td>0</td>
<td>52</td>
<td>1</td>
</tr>
<tr>
<td>10.0</td>
<td>0.403</td>
<td>500</td>
<td>18.84</td>
<td>0</td>
<td>52</td>
<td>1</td>
</tr>
<tr>
<td>5.0</td>
<td>0.746</td>
<td>1000</td>
<td>18.84</td>
<td>0.1504</td>
<td>45.9</td>
<td>6.24</td>
</tr>
<tr>
<td>10.0</td>
<td>0.403</td>
<td>1000</td>
<td>18.84</td>
<td>0.1504</td>
<td>45.9</td>
<td>4.62</td>
</tr>
<tr>
<td>5.0</td>
<td>0.746</td>
<td>1500</td>
<td>18.84</td>
<td>0.555</td>
<td>43.9</td>
<td>9.38</td>
</tr>
<tr>
<td>10.0</td>
<td>0.403</td>
<td>1500</td>
<td>18.84</td>
<td>0.555</td>
<td>43.9</td>
<td>6.91</td>
</tr>
<tr>
<td>5.0</td>
<td>0.746</td>
<td>2000</td>
<td>18.84</td>
<td>1.122</td>
<td>42.8</td>
<td>13.67</td>
</tr>
<tr>
<td>10.0</td>
<td>0.403</td>
<td>2000</td>
<td>18.84</td>
<td>1.122</td>
<td>42.8</td>
<td>10.06</td>
</tr>
</tbody>
</table>

*Ice mass fraction \( \alpha \), is 0.002.*
implementing the turbulence model in the following section. The impacts at the stagnation point produce ejecta that travel downstream because their path angles are large. But at the same time, the impacts at a downstream point send a portion of their ejecta upstream toward and beyond the stagnation point.

IV. Application of Turbulence Model

In [12], Hove and Shih recognized that the kinetic energy of the ice particles in the freestream is equivalent to turbulence energy in the shock layer. The presence of ice particles in the freestream is thus equivalent to having a turbulent freestream flow. Hove and Shih [12] point out that such an increase in the effective turbulence level cannot increase the heat transfer rate in the stagnation region; the flow in the stagnation region moves so slowly that the boundary layer tends to become laminar even when the freestream flow is turbulent. The explanation of the enhanced heat transfer rate offered by Hove and Shih was that the surface roughness caused by the ice-particle impacts produced the enhancement. Rough surfaces are known to increase the wall heat transfer rate greatly in most cases. However, this explanation is invalid in the stagnation region; it is known that the stagnation-point flow is always laminar even when the surface is rough, provided there is no mass flow injection from the wall [11]. However, there is a valid explanation of the heat transfer rate enhancement that has never been invoked before. The issuing of fragments from the wall after impact is a form of mass injection from the wall. In [11], Park theorized that an injection from the stagnation-point wall induces turbulence and makes the boundary layer turbulent there. Several experimental evidences are given in that work to show that the boundary-layer flow is indeed turbulent in the presence of wall injection.

A. Description of Existing Injection-Induced Turbulence Model

According to Park’s theory [11], the turbulent viscosity at wall \( \nu_w \) is expressed as

\[
\nu_w = 0.4 \dot{m} \delta
\]

where \( \dot{m} \) is the mass flow rate of injection \( \dot{m} = \chi \rho_w V_w \). The mixing length \( \delta \) is given by

\[
\delta = \delta_{\text{max}} \left[ 1 - \exp(-\tau v_i/\delta_{\text{max}}) \right]
\]

where \( \delta_{\text{max}} \) is the equivalent injection velocity, which, in the present case, becomes

\[
\delta_{\text{max}} = \max \left[ \frac{35}{2} \frac{\rho_i \mu}{(d/ds)} \sqrt{0.1 (\rho/250)^{1/2}} \right]
\]

and

\[
\frac{du}{ds} = \frac{V_a}{R} \left( \frac{2 \rho_s}{\rho} \right)
\]

Here, \( v_i \) is the equivalent injection velocity, which, in the present case, becomes

\[
v_i = \chi \rho_w V_w/\rho_i
\]

The time parameter \( \tau \) was determined empirically to fit the existing ablation data.

In a flow not containing particles, this turbulence energy at the wall decays in the boundary layer by the natural decay process such as that described by the \( k-\varepsilon \) model [11]. However, in the present case of ice-particle impact, all fragments remain in the boundary layer. Therefore, the turbulent intensity created at the wall \( \nu_w \) will not decay. Thus, the wall heat transfer rate in this case can be calculated by replacing the viscosity \( \mu \) in the well-known heat transfer rate equation by the effective (combined) viscosity \( \mu + \nu_w \). Because heat transfer rate is proportional to the square root of viscosity, the enhancement ratio \( q_i/q \) is simply

\[
\frac{q_i}{q} = \sqrt{\frac{\mu + \nu_w}{\mu}}
\]

This relationship holds even when the heat transfer occurs mostly in the form of surface recombination, because the coefficients of effective species diffusion are proportional to the effective viscosity.

B. New Turbulent Model for Impacting Environment

An improvement to the theory in [11] is proposed in the present work by using the fact that an ice-particle impact produces a crater. This method is tentatively named the crater-induced turbulence (CIT) model. In flight, even though ice-particle impacts occur at finite intervals, the phenomenon is cumulative. As a result, the heatshield surface will be mostly covered by impact craters. Each crater will produce a local vortex flow motion as shown in Fig. 8. Therefore, it would seem more appropriate to relate the turbulence mixing length to the dimensions of the crater. It is proposed that the mixing length in Eq. (20) is expressible as

\[
\delta_{\text{mix}} = \alpha \times \text{crater depth}
\]

where the crater depth is given by Eq. (2).

The CIT model still uses the time constant \( \tau \) in Eq. (19). The constant of proportionality \( \alpha \) and the time constant \( \tau \) are determined by comparing with the experimental data of Dunbar obtained in a wind tunnel, as given below.
The α and τ values can both be determined by demanding that Dunbar et al.’s experimental data are reproduced at more than one data point. The α and τ values so determined are used in calculating the heat transfer rate to the stagnation point in the presence of ice-particle impacts. By dividing this heat transfer rate by that without ice-particle impacts, the enhance ratio \( q_i/q \) [see Eq. (23)] is determined.

The enhancement factor is calculated for the representative wind-tunnel test case \( \rho_\infty = 0.0537 \text{ kg/m}^3 \) and \( U_\infty = 1100 \text{ m/s} \) using \( \alpha = 5.0 \) and \( \tau = 5 \times 10^{-3} \). The results are presented in Fig. 9.

In Fig. 10, calculation is made for the 1100 m/s case with \( \rho_\infty \) values of 0.0313, 0.0537, and 0.0692 kg/m\(^3\). As seen in Fig. 10, there is a fair agreement between the present calculation and the wind-tunnel data.

D. Application of the Crater-Induced Turbulence Model to the Experimental Condition

The above procedure leads to the expression for the heat transfer rate in the presence of ice-particle impacts

\[
\frac{q_i}{q} = \sqrt{\frac{\mu + \nu_i}{\mu}}
\]

(28)

where \( \nu_i \) is the turbulent viscosity at the wall given by Eq. (18), \( \nu_i = 0.4 \rho_i \delta \). Here, \( \delta \) is the mass flow rate of injection \( \dot{m} = \chi \rho_\infty V_a \).

This mass flow rate is determined using the experimental results obtained by present authors and calculation results of Sec. III. The mixing length \( \delta \) is given by Eq. (19),

\[
\delta = \delta_{\text{max}} = 3 \left( \frac{3}{2 \pi} \times \frac{\Delta M}{\rho_i} \right)^{1/3}
\]

(29)

This maximum mixing length \( \delta_{\text{max}} \) is determined using the experimental results obtained by the present authors, Eq. (2). Using the \( \alpha \) value of 5.0 and the \( \tau \) value of \( 5 \times 10^{-3} \) sec, which are determined from Dunbar’s study as explained in the previous chapter, the heat transfer rate enhancement factor was calculated. In doing so, ice mass fraction \( \chi \) is assumed as 0.002. This mass fraction is deduced from the flight measurement in [1–3], and is the average mass fraction of ice in the atmosphere. The \( \beta \) values are calculated by using the experimental results obtained by the present authors, Eq. (1), and \( \sigma \) values are calculated by Eq. (4).

The results are presented in Table 2 and Fig. 11 for several representative flight conditions. As can be seen in the table and the figure, there is no heat transfer enhancement when the flight velocity is 500 m/s. This is because craters are not formed at the vehicle’s surface. But when the flight velocity is greater than 1000 m/s, the

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C. Application of the Crater-Induced Turbulence Model to Dunbar et al.’s Results

In [9], Dunbar et al. measured heat flux increase by particle impacts in a wind tunnel. Solid particles (magnesium oxide, silicone carbide, and glass particles) impacted the graphite and metallic materials, such as titanium, inconel, stainless steel, and platinum in their experiment. They plotted the stagnation-point heat transfer rate values obtained using the Stanton number

\[
St = \frac{\text{Heat transfer rate}}{\rho_\infty U_\infty (H_a - H_w)}
\]

(25)

This result is reproduced in Fig. 9. Here, the abscissa \( \chi = \chi_{\alpha}(1 + \beta) \) is the ratio of the mass flux of fragments rebounding from the target wall to the mass flux of the air flow. The fragments consist of the projectile particles and the target material. As seen in the figure, the heat transfer rate increases by several factors due to the particle impacts in the tested range of conditions.

The data in Fig. 9 cover the stagnation-point pressure values from about 0.2 to about 2 atm, and enthalpy values from about 0.28 to 4.2 MJ/kg. The freestream velocities are in the range from 760 to 1700 m/s, and the freestream densities are in the range from 0.0312 to 0.0692 kg/m\(^3\). These values are used in calculating the heat flux enhancement using the CIT model. In [9], Dunbar et al. fitted the wind-tunnel data by an expression

\[
St = 0.098 \chi^{0.317}
\]

(26)

This relation between the particle loading and heat transfer rate deduced by Dunbar et al. is used in the present work.

In [18,19], an impact experiment was conducted against metallic targets and graphite targets. The relationship between the depths of the created craters, impact velocity, and projectile diameter observed in their experiment is fitted in the present work by

\[
\delta_{\alpha} = \alpha \times (0.835 + 0.00251 \times V_a) \times \text{projectile diameter}
\]

for metallic target

\[
\delta_{\alpha} = \alpha \times (1.073 + 0.00226 \times V_a) \times \text{projectile diameter}
\]

for graphite target

(27)

Because Dunbar et al.’s experiment used metallic and graphite targets, these crater depth values are substituted in Eq. (19) to determine the mixing length for Dunbar et al.’s [9] test conditions. In doing so, it became necessary to know the value of \( \chi \) in the absence of impacts. The lowest value in Dunbar et al.’s experimental data \( \chi = 4 \times 10^{-4} \) was considered to be the condition of no ice-particle impacts.
heat transfer rates are higher. At the altitude of 5 and 10 km, the enhancement factor becomes up to about 10 and 14, respectively.

V. Conclusions

In the present work, the authors’ earlier experimental work is summarized first. From those results, trajectories of fragments were calculated. Fragments are only slightly vaporized and stay as solid particles in the stagnation region. These fragments can produce a turbulent flow in the stagnation region. When the flow becomes turbulent, the heat transfer rate increases. The enhancement of heat transfer rate is determined by a new model tentatively named the CIT model. In this model, an assumption was made that the turbulent mixing length in the model is proportional to the depth of the impact craters. The constant of proportionality was determined from the existing experimental data taken in wind-tunnel tests. The model is used in determining the heat flux enhancement in the stagnation region. Sample calculations of the heat transfer rate enhancement factor so derived were made for several representative flight conditions of high-speed vehicles. It is shown that the heat transfer rate may increase up to 14 times that without ice-particle impacts in an icing environment.

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