Modeling of Ice Particle Impacts on a Composite Material

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This paper examines what would happen if a hypersonic vehicle flies through an atmosphere laden with ice particles. In a previous research of the present authors, an experiment was performed to determine the characteristics of ice particle impact phenomenon. Therein, the mass loss from vehicle’s surface material by the impacts was measured and the fragments’ behavior was studied. In the present work, the trajectories of fragments from the stagnation region were calculated at the experimental condition. It was found that the flow in the stagnation region is turbulent. Turbulent flow increases the heat transfer rate to the surface, and consequently the mass loss increases. To determine the extent of heat transfer rate increase, the new turbulence model, tentatively named crater-induced turbulence model, was proposed. Therein, an assumption was introduced that the turbulent mixing length is proportional to the depth of the impact craters. The constant of proportionality was determined from the existing experimental data taken in wind tunnel tests. It is shown that heat transfer rate may increase up to 10.20 times that without ice particle impacts.

Nomenclature

\( c_1, c_2, c_3 \) Non-dimensional coefficient for fragments’ mass distribution, Eq. (3).

\( c_5 \) Non-dimensional coefficient for fragments’ velocity distribution, Eq. (6).

\( D \) Fragments’ diameter.

\( E \) Projectile kinetic energy, J.

\( f \) Stream function, Eq. (11).

\( F \) Distribution function, Eq. (6).

\( H \) Flow enthalpy, J/kg.

\( H_v \) Heat of conversion of 1g of ice at 0 °C to steam at 100 °C, 3012 J/kg.

\( \Delta M \) Displaced mass, g.

\( m \) Mass of a particle.

\( q \) Stagnation-point heat transfer rate without ice impact.

\( q_i \) Stagnation-point heat transfer rate with ice impact.

\( q_{st} \) Stagnation point heat flux.

\( R \) Nose radius.

\( Re \) Reynolds number.

\( St \) Stanton number.

\( t \) Time.

\( T \) Temperature, K.

\( u \) Velocity in x direction.

\( v_i \) Equivalent injection velocity.

\( v \) Velocity in y direction.

\( V \) Magnitude of velocity, \( \sqrt{u^2 + v^2} \).

\( x \) Horizontal distance (Fig. 4).

\( y \) Vertical distance (Fig. 4).

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HEN a high speed vehicle flies through a cloud of rain drops, snowflakes, or hail, damage may occur to
the vehicle. Although hail is rare, rain drops exist as ice particles at high altitudes [1–3]. The impacts of such
ice particles on a space vehicle are a concern worthy of study. An environment, in which solid or liquid
particles impact a flying object, is known as erosive environment. The impacts increase the wearing rate of the
heatshield material by two mechanisms: direct gouging of material by forming impact craters and increase in
thermochemical ablation rate by an increase in heat transfer rates. A considerable amount of research has been
carried out on this subject [4, 5]. However, the results of such studies have not yet been reported in whole, at least in
open literature.

The phenomenon of impact of a solid object with a solid surface at a high speed has been studied and reported in
open literature in connection with the astronomical impacts [6]. It is well known that, when an asteroid impacts a
planetary surface, a crater is formed. In the cratering process, the impacting body is fragmented and, when the
impact energy is sufficient, it is vaporized. The target surface loses mass by forming a crater. The mass ejected from
the surface is also fragmented and sometimes vaporized.

A theoretical model [7] to describe this cratering phenomenon should predict the quantity of the mass removed
from the surface and the momentum and energy imparted to the surface. In order to develop a theoretical model to
describe the ice-particle impact phenomenon, the quantity of the mass removed from the surface, the magnitudes of
the momentum and energy carried by the issuing mass, and the direction of the issuing mass were measured
experimentally [8] by the present authors. The results of this study provide a formula which could be used in
determining the surface mass loss rate due to direct gouging.

The behavior of the issuing mass has an impact on the second mechanism of mass removal, i.e., increase in heat
transfer rate. It has been observed that, in an erosive environment, the bow shock wave formed over a blunt body
bulges forward, and shock layer pressure and heat transfer over the body increase greatly [9,10]. The exact
relationship between the mass, momentum and energy of the issuing mass and the extent of increase in heat transfer
is not known presently. However, it is suspected that the increase in the heat transfer is because of the turbulence
generated by the fragments. [11, 12]

The objective of the present work is to study the consequence of ice particle impacts on heat transfer

\[ \alpha_c \] Fraction of ice mass in the atmosphere.
\[ \beta \] Ratio of the target material mass removed by impact to the mass of impacting ice particle.
\[ \delta \] Turbulent mixing length.
\[ \phi \] Inclination angle of the ejecta with respect to the normal.
\[ \Phi \] Heat flux blockage factor.
\[ \gamma \] Non-dimensional exponent, Eq. (7).
\[ \mu \] Viscosity, N-s/m².
\[ \nu \] Turbulent viscosity, N-s/m².
\[ \rho \] Density.
\[ \sigma \] Scale parameter, Eq. (4).
\[ \tau \] Injection-induced turbulence parameter, sec, Eq. (19).
\[ \chi \] Fragment mass-to-air mass ratio, \( \alpha_c (1+\beta) \).
\[ \omega \] Half width angle, deg., Eq. (6).

Subscripts
\[ a \] Ambient condition.
\[ D \] Size.
\[ i \] Impact.
\[ m \] Mass.
\[ max \] Maximum value.
\[ min \] Minimum value.
\[ rel \] Relative value.
\[ s \] Shock layer.
\[ t \] Target material
\[ v \] Velocity.
\[ w \] Wall.
\[ \phi \] Ejecta angle.
phenomenon. A composite material is chosen here because it is commonly used as a structural material for high speed vehicles. A new turbulence model, a modified version of the injection-induced turbulent model [11], is used in describing this heating environment. An assumption was introduced that the turbulent mixing length is proportional to the depth of the impact craters. The constant of proportionality was determined from the existing experimental data taken in wind tunnel tests. The model is used in determining the heat flux enhancement in the stagnation region.

II. Summarization of Earlier Experimental Results

The heatshield of a reentry vehicle is made in many cases with a composite material. One of those composite materials is carbon/epoxy. In Ref. [8], the effect of ice particle impacts on a carbon/epoxy was studied experimentally. An ice projectile was accelerated by a two-stage light gas gun. The Mach number of the projectile ranged from Mach 2 to 3. A carbon/epoxy composite material was prepared for the target specimen. Test sets and results are summarized in Table 1.

In that experiment, a total mass removed from target specimen was measured to be expressible as

$$\Delta M = -0.00327 + 0.000648 \times E.$$  

(1)

In the above equation, $\Delta M$ and $E$ denote removed mass in g and projectile kinetic energy in J. From the above equation, crater size was determined as

$$d = \left( \frac{3}{2\pi} \frac{\Delta M}{\rho_t} \right)^{1/3}$$

(2)

where, $d$ is the equivalent crater size.

From here on, attention will be focused on the increase in heat transfer rate due to particle impacts. An impact produces fragments. The issuing fragments consist of the broken-up pieces of the ice projectile and target material in a powered form. The behavior of ice fragments was obtained from Ref. [13]. The behavior of target material was studied using a witness film and a high-speed camera. Figure 1 shows the issuing mass distribution for Tests 08071702, 08071703, and 08071802. From Fig. 1, the mass distribution curve was derived as

$$N(>V) = c_1 c_2 \exp \left( -c_2 \frac{V}{V_0} \right) + (1 - c_1) c_3 \exp \left( -c_3 \frac{V}{V_0} \right)$$

(3)

where, the constants $c_1$, $c_2$, and $c_3$ are 0.2452, 26820, and 4118, respectively. $V_0$, a scale parameter related to the total volume, is $5.332 \times 10^7 \mu$m$^3$. Scale parameters of ice fragments, $\sigma_i$, and of target, $\sigma_t$, are 52 $\mu$m and 35.15 $\mu$m, respectively. From these values the representative characteristic length, $\sigma$, was determined as

$$\sigma = \frac{1}{\left( \frac{V_{ice}}{V_{ice} + V_{target}} \right)^{1/3} + \left( \frac{V_{target}}{V_{ice} + V_{target}} \right)^{1/3}} \left[ \left( \frac{V_{ice}}{V_{ice} + V_{target}} \right)^{1/3} \sigma_t + \left( \frac{V_{target}}{V_{ice} + V_{target}} \right)^{1/3} \sigma_i \right]$$

(4)

where $V_{ice}$ and $V_{target}$ are volumes of fragments of ice projectile and target material, respectively.

The directional distribution of solid particles is shown in Fig. 2. Ejecta angle, $\phi$, is the direction of ejecta fragments, i.e., inclination angle with respect to the normal direction. From this figure, it can be found that many solid particles are issued at high angles.

The fragments’ velocity was also empirically formulated in Ref. [8] as

$$V(D, \phi) = V_i \cdot F_{i, \phi}(\phi) \cdot F_{i, \phi}(D)$$

(5)

where $V$ is the fragments’ velocity and $V_i$ is the projectile velocity. Here, $F_{i, \phi}(\phi)$ represents the effect of ejecta angle. Figure 3 shows the fragments’ velocity divided by the projectile velocity in the ejecta angle region for several tests. The line fitted to the data is

$$F_{i, \phi}(\phi) = c_5 \exp \left( -\frac{\phi - \phi_{\phi, \text{max}}}{2 \omega_\phi} \right)^2$$

(6)

where constant $c_5$ is 1.2152. And $\phi_{\phi, \text{max}}$, the angle where the maximum fragment velocity occurs, and $\omega_\phi$, the half width, are 82.12$^0$ and 11.706$^0$, respectively.

In Eq. (5), $F_{i, \phi}(D)$ represents the effect of fragments’ size, $D$,
\[ F_{v,D}(D) = \left( \frac{D}{D_{\text{min}}} \right)^\gamma \]  

where \( D_{\text{min}} \) and exponent \( \gamma \) are 1 \( \mu \text{m} \) and 0.366, respectively.

### Table 1. Ice impact test sets for specimens

<table>
<thead>
<tr>
<th>Test #</th>
<th>Energy (J)</th>
<th>Displaced Mass (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>08042501</td>
<td>6.812</td>
<td>N/A</td>
</tr>
<tr>
<td>08042502</td>
<td>6.516</td>
<td>N/A</td>
</tr>
<tr>
<td>08042503</td>
<td>8.224</td>
<td>N/A</td>
</tr>
<tr>
<td>08042506</td>
<td>9.593</td>
<td>N/A</td>
</tr>
<tr>
<td>08042507</td>
<td>7.647</td>
<td>N/A</td>
</tr>
<tr>
<td>08042508</td>
<td>6.107</td>
<td>0.0002</td>
</tr>
<tr>
<td>08042601</td>
<td>8.124</td>
<td>0.0018</td>
</tr>
<tr>
<td>08042602</td>
<td>8.538</td>
<td>0.0025</td>
</tr>
<tr>
<td>08042604</td>
<td>9.988</td>
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<td>08042605</td>
<td>9.342</td>
<td>0.0026</td>
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<td>08050901</td>
<td>6.972</td>
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<td>08050902</td>
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<td>10.313</td>
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<tr>
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<td>8.608</td>
<td>0.0024</td>
</tr>
<tr>
<td>08051102</td>
<td>6.002</td>
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<td>08051103</td>
<td>9.778</td>
<td>0.003</td>
</tr>
<tr>
<td>08051202</td>
<td>6.820</td>
<td>0.0016</td>
</tr>
<tr>
<td>08062701</td>
<td>7.292</td>
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</tr>
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<td>08062902</td>
<td>5.762</td>
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<td>08063002</td>
<td>10.598</td>
<td>0.0034</td>
</tr>
<tr>
<td>08071702</td>
<td>6.820</td>
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</tr>
<tr>
<td>08071703</td>
<td>6.820</td>
<td>N/A</td>
</tr>
<tr>
<td>08071802</td>
<td>8.004</td>
<td>N/A</td>
</tr>
</tbody>
</table>

**Fig. 1** Cumulated number larger than volume, \( V \), for Test 08071702, 08071703, and 08071802.

**Fig. 2** Relationship between ejected mass and ejecta angle for Test 08071702, 08071703, and 08071802. The open symbols mean the experimental data, and the horizontal bar indicates the measured angle interval.
III. Motion of Rebounding Fragments

A. Governing Equations

Motion of solid particles issuing from the heat shield wall was studied in Ref. [14-16]. In those studies, the issuing particles were assumed to be spherical in shape and so the drag coefficient was assumed to be unity, which is the Newtonian value. This assumption was made here also. The density of the fragments was assumed to be that of ice, i.e. 0.919 g/cm³.

To simplify the problem, only the stagnation region flow, sketched in Fig. 4, was considered here. The motion of an issuing fragment was dictated by the Newton’s equation of motion. The force acting on a particle is the drag. It was calculated as a vectorial sum of the force due to flow motion and force due to particle motion. The angle between the flow and the particle is described by $\theta$. The x- and y- components of the drag force produce the acceleration

\[
\frac{m}{dt} = -\text{drag} \times \cos(\theta) \\
\frac{m}{dt} = -\text{drag} \times \sin(\theta)
\]

(8)

The particle is heated on the windward side. Total heat transfer to the particle is calculated as the product of the stagnation-point heat transfer rate and the frontal surface area $\pi(D/2)^2$. In the calculation of the stagnation-point heat transfer rate, a simplification $\rho_\mu = \text{constant}$, is made. Using Goulard’s analysis and approximating the parameter $\sqrt{\rho_s/\rho_a}$ to be $\sqrt{18}$, the Goulard’s formula [17] for stagnation-point heat transfer rate $q_u$ becomes

\[
q_u = 0.590 \rho V_{rel} (H - H_u)/\sqrt{\text{Re}}
\]

(9)

where the density $\rho$ is $\rho_s$ when the particle is inside the shock layer and $\rho_a$ when it is outside of the shock layer. The Reynolds number Re is

\[
\text{Re} = \frac{\rho V_{rel} (D/2)}{\mu_u}.
\]

(10)
The flow and wall enthalpy difference \( H - H_w \) is \( V_{rel}^2/2 + 1006 \times (T - T_w) \) in \( J/kg \).

Because of vaporization, the heat transfer rate is convectively blocked partly. By defining the stream function \( f \) by

\[
f = -\frac{\rho v}{\sqrt{2\mu \rho_s (du/dx)}}
\]

the heat transfer rate to a vaporizing surface is the value given by Eq. (9) times the blockage factor \( \Phi \), which is smaller than 1 and is a function of the wall value of \( f \). The acceleration parameter \( du/dx \) is given by

\[
\frac{du}{dx} = \frac{V_{rel}}{D/2} \sqrt{2\rho_s/\rho_s}.
\]

Numerical solution of the heat transfer problem, such as that in Ref. 16, shows that \( \Phi \) is approximately a linear function of \( f_w \) in the small range of \( f_w \), as

\[
\Phi = 1 - f_w.
\]

The mass flow rate of the vapor at the stagnation point of a fragment, \( \rho v \), appearing in Eq. (11), is calculated in general by dividing the heat transfer rate value \( q_{st} \) by the heat of vaporization \( H_v = 3012 J/kg \).

\[
\Phi = 1 - \frac{q_{st}/H_v}{\sqrt{2\rho_s/\rho_s (du/dx)}_w}.
\]

Therefore, the blockage-corrected stagnation-point heat transfer rate \( q_{st} \) satisfies the equation

\[
q_{st} = 0.590 \rho V_{rel} (H - H_w) \frac{1 - \frac{q_{st}/H_v}{\sqrt{2\rho_s/\rho_s (du/dx)}}}{\sqrt{Re}}.
\]

By solving above equation for \( q_{st} \), one obtains

\[
q_{st} = \frac{0.590 \rho V_{rel} (H - H_w) / \sqrt{Re}}{1 + 0.590(h - h_w)/H_v}.
\]

By dividing this \( q_{st} \) by \( H_v \), one obtains the rate of vaporization of the fragment. Thus the equation governing the mass of the fragment is

\[
\frac{dm}{dt} = q_{st} \times \pi \left(\frac{D}{2}\right)^2.
\]

B. Calculated Fragments’ Behavior in Shock Layer

In Fig. 5, the trajectories of the rebounding fragments are shown for those issuing from the ejecta angle \( \Phi \) of 70° for the experimental condition, i.e., mass of ice particle = 0.01884 g and impact velocity = 1020 m/s, at 5 km altitude as listed in Table 2. The issuing velocity of each fragment is given by Eq. (5). Sizes of the fragments ranging from \( D/\sigma = 0.3 \) to 6.0 are shown. One notices in this figure that all particles turn back and reach the wall.

Table 2 Selected flow environment.

<table>
<thead>
<tr>
<th>Altitude, km</th>
<th>Density, kg/m³</th>
<th>Flight velocity, m/s</th>
<th>Ice projectile mass, mg</th>
<th>( \beta )</th>
<th>( \sigma, \mu m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.05</td>
<td>0.742</td>
<td>1020</td>
<td>18.84</td>
<td>0.1635</td>
<td>45.78</td>
</tr>
</tbody>
</table>

Figure 6 shows the mass variation in the shock layer in detail. As mentioned in the previous section, the most fragments are issuing at high ejecta angles (50 deg – 80 deg). Therefore, only the higher ejecta angle fragments are considered. The returning size of a fragment is larger than 96 % of the initial size of that fragment. From this figure, it can be said that most fragments survive in the shock layer.

The trajectories of \( D/\sigma = 1.136 \) are shown for different ejecta angles in Fig. 7. As mentioned before, this figure also considered the only high ejecta angle fragments.

From Refs. [10–12], it is known that an erosive environment makes flow turbulent in the stagnation region. From above results, it can be said that the ice particle impact can also make flow turbulent because all fragments still exist in the stagnation region even though they are partly vaporized. The impacts at the stagnation point produce ejecta that travel downstream because their path angles are large. But at the same time, the impacts at a downstream
point send a portion of their ejecta upstream toward and beyond the stagnation point.

IV. Application of Turbulence Model

In Ref. [12], Hove and Shih recognized that the kinetic energy of the ice particles in the freestream is equivalent to turbulence energy in the shock layer. The presence of ice particles in the freestream is thus equivalent to having a turbulent freestream flow. Ref. [12] points out that such an increase in the effective turbulence level cannot increase the heat transfer rate in the stagnation region: the flow in the stagnation region moves so slow that the boundary layer tends to become laminar even when the freestream flow is turbulent. The explanation of the enhanced heat transfer rate offered by Hove and Shih was that the surface roughness caused by the ice particle impacts produced the enhancement. Rough surfaces are known to increase the wall heat transfer rate greatly in most cases. However, this explanation is invalid in the stagnation region: it is known that the stagnation–point flow is always laminar even when the surface is rough, provided there is no mass flow injection from the wall [11].

However, there is a valid explanation of the heat transfer rate enhancement that has never before invoked. The issuing of fragments from the wall after impact is a form of mass injection from the wall. In Ref. [11], Park theorized that an injection from the stagnation-point wall induces turbulence, and makes the boundary layer turbulent there. Several experimental evidences are given in that work to show the boundary layer flow is indeed turbulent in the presence of wall injection.

A. Description of Existing Injection Induced Turbulence Model

According to Park’s theory [11], the turbulent viscosity at wall $\nu_w$ is expressed as
\[ \delta = \delta_{\text{max}} \left[ 1 - \exp\left(-\nu_i / \delta_{\text{max}} \right) \right] \]  

(19)

where \( \delta_{\text{max}} \) is given by

\[ \delta_{\text{max}} = \max \left[ 35 \sqrt{\frac{\rho_s \mu_s}{2(du/ds)}} \right] \]  

(20)

and

\[ \frac{du}{ds} = \frac{V_a}{R} \sqrt{\frac{2 \rho_s}{\rho_s}} \]  

(21)

Here, \( \nu_i \) is the equivalent injection velocity, which, in the present case, becomes

\[ \nu_i = \nu_{\text{eq}} \]  

(22)

The time parameter \( \tau \) was determined empirically to fit the existing ablation data.

In a flow not containing particles, this turbulence energy at wall decays in the boundary layer by the natural decay process such as that described by the \( k-\varepsilon \) model. [11] However, in the present case of ice particle impact, all fragments remain in the boundary layer. Therefore, the turbulent intensity created at wall, \( \nu_w \), will not decay. Thus, the wall heat transfer rate in this case can be calculated by replacing the viscosity \( \mu \) in the well-known heat transfer rate equation by the effective (combined) viscosity \( \mu + \nu_w \). Because heat transfer rate is proportional to the square root of viscosity, the enhancement ratio \( q_i/q \) is simply

\[ \frac{q_i}{q} = \sqrt{\frac{\mu + \nu_w}{\mu}} \]  

(23)

This relationship holds even when the heat transfer occurs mostly in the form of surface recombination, because the coefficients of effective species diffusion are proportional to the effective viscosity.

**B. A New Turbulent Model for Impacting Environment**

An improvement to the theory in Ref. [11] is proposed in the present work by utilizing the fact that an ice particle impact produces a crater. This method is tentatively named crater-induced turbulence (CIT) model. In flight, even though ice particle impacts occur at finite intervals, the phenomenon is cumulative. As a result, the heatshield surface will be mostly covered by impact craters. Each crater will produce a local vortex flow motion as shown in Fig. 8. Therefore, it would seem more appropriate to relate the turbulence mixing length to the dimensions of the crater. It is proposed that the mixing length in Eq. (20) is expressible as

\[ \delta_{\text{cm}} = \alpha \times \text{crater depth} \]  

(24)

where the crater depth is given by Eq. (2).

The CIT model still uses the time constant \( \tau \) in Eq. (19). The constant of proportionality \( \alpha \) and the time constant \( \tau \) are determined by comparing with the experimental data of Dunbar obtained in a wind tunnel, as given below.

**C. Application of CIT Model to Dunbar et al.’s Results**

In Ref. 9, Dunbar et al. measured heat flux increase by particle impacts in a wind tunnel. Solid particles (Magnesium oxide, Silicone carbide and glass particles) impacted the graphite and metallic materials, such as titanium, inconel, stainless steel and platinum in their experiment. They plotted the stagnation-point heat transfer rate values obtained using the Stanton number \( St \)
\[
St = \frac{\text{Heat transfer rate}}{\rho_a U_a (H_a - H_u)}.
\]

This result is reproduced in Fig. 9. Here, the abscissa \( \chi = \alpha_c (1 + \beta) \) is the ratio of the mass flux of fragments rebounding from the target wall to the air mass flux. The targets consist of the fragments of the projectile particles and the target material. As seen in the figure, heat transfer rate increases by a factor of several due to the particle impacts in the tested range of conditions.

The data in Fig. 9 cover the stagnation-point pressure values from about 0.2 to about 2 atm, and enthalpy values from about 0.28 to 4.2 MJ/kg. The freestream velocities are in the range from 760 m/s to 1700 m/s, and the freestream densities are in the range from 0.0312 to 0.0692 kg/m\(^3\). These values are used in calculating the heat flux enhancement using CIT model. In Ref. 9, Dunbar et al. fitted the wind tunnel data by an expression

\[
St = 0.098 \chi^{0.317}.
\]

In Refs. [18] and [19], an impact experiment was conducted against metallic targets and graphite targets. The relationship between the depths of the created craters, impact velocity, and projectile diameter observed in their experiment is fitted in the present work by

\[
\delta_m = \alpha \times (0.835 + 0.00251 \times V_i) \times \text{projectile diameter} \quad \text{for metallic target}
\]

\[
\delta_m = \alpha \times (1.073 + 0.00226 \times V_i) \times \text{projectile diameter} \quad \text{for graphite target}
\]

Because Dunbar et al.’s experiment used metallic and graphite targets, these crater depth values are substituted in Eq. (19) to determine the mixing length for Dunbar et al.’s test conditions. In doing so, it became necessary to know the value of \( \chi \) in the absence of impacts. The lowest value in Dunbar et al.’s experimental data, \( \chi = 4 \times 10^{-4} \), was considered to be the condition of no ice particle impacts.

The \( \alpha \) and \( \tau \) values can both be determined by demanding that Dunbar et al.’s experimental data are reproduced at more than one data points. The \( \alpha \) and \( \tau \) values so determined are used in calculating the heat transfer rate to the stagnation point in the presence of ice particle impacts. By dividing this heat transfer rate by that without ice particle impacts, the enhance ratio \( q_i/q \) (see Eq. (23)) is determined.

The enhancement factor is calculated for the representative wind tunnel test case \( \rho_a = 0.0537 \text{ kg/m}^3 \) and \( U_a = 1100 \text{ m/sec} \) using \( \alpha = 5.0 \) and \( \tau = 5 \times 10^{-3} \). The results are presented in Fig. 9.

![Fig. 9 Comparison of Stanton number between the injection-induced turbulence model and wind tunnel data for the representative wind tunnel condition.](image)

![Fig. 10 Comparison of Stanton number between the crater-induced turbulence model and wind tunnel data for \( U_a = 1100 \text{ m/sec} \) and three \( \rho_a \) values.](image)

In Fig. 10, calculation is made for the 1100 m/s case with \( \rho_a \) values of 0.0313, 0.0537 and 0.0692 kg/m\(^3\). As seen in Fig. 10, there is a fair agreement between the present calculation and the wind tunnel data.
D. Application of CIT Model to the Experimental Condition

The above procedure leads to the expression for the heat transfer rate in the presence of ice particle impacts:

\[
\frac{q_i}{q} = \sqrt{\frac{\mu + \nu_w}{\mu}}
\]  

(28)

where \( \nu_w \) is turbulent viscosity at wall is given by

\[
\nu_w = 0.4 \bar{m} \delta
\]  

(29)

where \( \bar{m} \) is the mass flow rate of injection, \( \bar{m} = \bar{\rho}_i \bar{V}_i \). The mixing length \( \delta \) is given by

\[
\delta = \delta_{\text{max}} \left[ 1 - \exp\left(-\nu_i / \delta_{\text{max}}\right) \right]
\]  

(30)

where the maximum mixing length \( \delta_{\text{max}} \) is

\[
\delta_{\text{max}} = \alpha \frac{3}{2\pi} \frac{\Delta M}{\rho_i} \frac{\sqrt[3]{C}}
\]  

(31)

Using the \( \alpha \) value of 5.0 and the \( \tau \) value of \( 5 \times 10^{-3} \) sec, the heat transfer rate enhancement factor was calculated for the experimental condition given in Table 2. In doing so, \( \beta \) values are calculated by using the experimental results, Eq. (1) and \( \sigma \) values are calculated by Eq. (4).

The results are presented in Table 3. In Table 3, two values of ice mass fraction \( \alpha_c \) calculated, 0.002 and 0.01, are the average and maximum ice mass fraction given in Ref. [13], respectively. These average and maximum ice mass fraction are the average and largest ice mass fraction found in the atmosphere in the flight measurement reported in Refs. [1-3]. As can be seen in the table, in the average icing atmosphere, \( \alpha_c = 0.002 \), heat transfer rates are enhanced up to a factor of about 4.65. In the extreme icing condition, \( \alpha_c = 0.01 \), the enhancement factor becomes up to about 10.20.

Table 3 Stagnation-point heat flux rate enhancement factor calculated by crater-induced turbulence model.

<table>
<thead>
<tr>
<th>( \alpha_c )</th>
<th>( \frac{q_i}{q} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.002 (average ice mass fraction)</td>
<td>4.65</td>
</tr>
<tr>
<td>0.01 (maximum ice mass fraction)</td>
<td>10.20</td>
</tr>
</tbody>
</table>

V. Conclusions

In the present work, first, the authors’ earlier experimental work is summarized. From those results, trajectories of fragments were calculated. Fragments are only slightly vaporized and stay as solid particles in the stagnation region. These fragments can produce a turbulent flow in the stagnation region. When the flow becomes turbulent, heat transfer rate increases. For the experimental condition considered, the enhancement of heat transfer rate is determined by a new model tentatively named crater-induced turbulence model. In this model, an assumption was made that the turbulent mixing length in the model is proportional to the depth of the impact craters. The constant of proportionality was determined from the existing experimental data taken in wind tunnel tests. The model is used in determining the heat flux enhancement in the stagnation region. It is shown that heat transfer rate may increase up to 10.20 times that without ice particle impacts.

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